Problem 1: Decide whether the following statements are true or false:

(a) If f, g are differentiable, then $\frac{d}{dx}(fg) = f'g'$.

(b) If a function is continuous, then it is differentiable.

(c) If a function is differentiable, then it is continuous.

(d) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist.

(e) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$.

(f) If $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ exist, then $\lim_{x\to a} f(x)$ exists.

(g) If $\lim_{x\to a} f(x) = M$ and $\lim_{x\to a} g(x) = N$, then $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{M}{N}$.

(h) Polynomials are everywhere continuous.

(i) Rational functions are continuous everywhere.

(j) A tangent line to a function $f(x)$ intersects the function only once.

(k) A tangent line to a function $f(x)$ cannot intersect the function infinitely many times.

(l) $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.

(m) All continuous functions have at least one x-value at which they are differentiable.

(n) All functions on $\mathbb{R}$ have a limit at some x-value in their domain.

(o) If f, g are differentiable, then $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)}{g'(x)}$.

(p) A tangent line to a function $f(x)$ at $x = a$ has the same value of $f(x)$ at $x = a$.

(q) Every function has a tangent line.

(r) If $g(x) < f(x)$ on (a, b), then $g'(x) < f'(x)$ on (a, b).

(s) If $h(x) = \frac{f(x)}{g(x)}$ and $g(a) = 0$, then $x = a$ has a vertical asymptote at $x = a$.

(t) If $\lim_{x\to\infty} f(x) = L$, i.e. x has a horizontal asymptote, then $\lim_{x\to-\infty} f(x) = L$.

(u) There is a function with a zero at $x = 0$ and a y-intercept of 6.

(v) If $f(x)$ is differentiable and decreasing on (a, b), then $f'(x) < 0$ on (a, b).

(w) If $f'(x) > 0$ on (a, b), then $f(x)$ is increasing on (a, b).

(x) If $f'(x) > 0$ on (a, b), then $f(x) > 0$ on (a, b).

(y) If $f(x)$ is continuous at $x = a$, then $\lim_{x\to a} f(x)$ exists.

(z) If $f(x), g(x)$ are continuous, then $\dfrac{f(x)}{g(x)}$ is continuous whenever it is defined.

Problem 2: Evaluate the following limits:

(a) $\lim_{x\to 1} \dfrac{3x^2+4x+1}{5x+7}$

(b) $\lim_{x\to 2}(4x-1)$

(c) $\lim_{x\to 3} \sin(\pi x)$

(d) $\lim_{x\to -7} \dfrac{\tan^3 x}{x-\pi}$

Problem 3: Give an example of the following:

(a) a continuous function (algebraically and graphically).

(b) a differentiable function.

(c) a function which is not differentiable (algebraically and graphically).

(d) a function whose limit exists (algebraically and graphically).

(e) a function whose limit does not exist (algebraically and graphically).

(f) a function whose left and right limits exist but whose limit does not exist.

(g) a function whose left and right limits are equal but whose limit does not exist.

(h) a function with a vertical asymptote.

(i) a function with a horizontal asymptote.

(j) a function with a zero.

(k) a function with no zeros.

(l) a function with no y-intercept.

(m) a function with a jump discontinuity.

(n) a function with an infinite discontinuity.

(o) a function with a removable discontinuity.

(p) a function with an infinite amount of zeros.

(q) a function with infinitely many infinite discontinuities.

(r) a function with infinitely many removable discontinuities.

(s) a polynomial with roots $x = -1, 2, 3$.

(t) a polynomial with roots $(-6, 0), (2, 0)$ and y-intercept $(0, 5)$.

(u) a function with y-intercept -2 and a zero at $x = 4$.

(v) a graph which is not the plot of a function.

(w) a graph which is not a function of x or y but is a function of some variable.

Problem 4: Define $f(x)$ to be the following function

$$f(x) = \begin{cases} 1 - x, & x \leq 1 \\ x^2 + ax + b, & x > 1 \end{cases}$$

Find values a, b so that $f(x)$ is everywhere continuous and differentiable.

Problem 5: Let $f(x) = x^2 + 5x - 1$. Find the average velocity of $f(x)$ on $[-1, 2]$. Use the definition of the derivative to find the instantaneous velocity of $f(x)$ at $x = 1$.

Problem 6: Find the following limit:

$$\lim_{x \to \infty} \sqrt{x^2 + 3x + 1} - 2x$$

Problem 7: Find the following limit:

$$\lim_{x \to \infty} \sqrt{x + 2} - \sqrt{x - 1}$$

Problem 8: Evaluate the following limit:

$$\lim_{x \to \infty} \ln(3x^2 - 4) - \ln(2x^2 + 1)$$

Problem 9: Define $f(x)$ to be the following function:

$$f(x) = \begin{cases} x^2 2^{-x}, & x \geq 0 \\ 4 - x, & x < 0 \end{cases}$$

Use the definition of $f(x)$ to find the following:

(a) $f(0)$

(b) $\lim_{x \to 2} f(x)$

(c) y-intercepts

(d) $\lim_{x \to 0^-} f(x)$

(e) x-intercepts

(f) $\lim_{x \to 0^+} f(x)$

(g) Classify any discontinuities for $f(x)$

(h) $\lim_{x \to 0} f(x)$

Problem 10: Define $f(x)$ to be the following function:

$$f(x) = \frac{(x+1)(2x-3)(x+2)}{(3x-7)(x+2)(x+3)}$$

(a) What is the y-intercept of $f(x)$?

(b) What are the x-intercepts of $f(x)$?

(c) What are the vertical asymptotes for $f(x)$?

(d) Where is $f(x)$ continuous?

(e) If $f(x)$ has any discontinuities, classify them.

(f) Identify any horizontal asymptotes $f(x)$ might have.

Problem 11: Evaluate the following limits:

(a) $\lim_{x\to 0^+} \ln x$

(b) $\lim_{x\to 2^+} \frac{x+6}{x-2}$

(c) $\lim_{x\to 1^-} \frac{x-4}{x+1}$

(d) $\lim_{x\to -2} \frac{2x+4}{x+2}$

(e) $\lim_{x\to 0} \frac{\cos x}{x}$

Problem 12: Use the definition of the derivative to find the derivative of the following functions:

(a) $1/x$

(b) $x^2 + 3x - 2$

(c) $1/x^2$

(d) $\sin x$

(e) $\cos x$

(f) $\sqrt{x+3}$

Problem 13: Calculate the following limits:

(a) $\lim_{x\to 0} \csc x - \cot x$

(b) $\lim_{x\to 0} \frac{3x}{\sin 5x}$

(c) $\lim_{x\to 0} \frac{\csc 7x}{\csc 5x}$

(d) $\lim_{x\to 0} \sin^2 3x$

(e) $\lim_{x\to 0} \frac{\tan x}{\sin x}$

(f) $\lim_{x\to 0} \frac{\sin^2(3x)}{x}$

(g) $\lim_{x\to 0} \frac{\tan x}{x}$

Problem 14: Evaluate the following limits:

(a) $\lim_{x\to\infty} \sin x$

(b) $\lim_{x\to\infty} \left(\frac{1}{2}\right)^x$

(c) $\lim_{x\to\infty} 7^x$

(d) $\lim_{x\to-\infty} \left(\frac{2}{3}\right)^x$

(e) $\lim_{x\to\infty} \frac{3x^2+4x-5}{4x^2-3x+5}$

(f) $\lim_{x\to\infty} \frac{16x^2+17x+12}{x^3-2x^2-14x+1}$

(g) $\lim_{x\to\infty} \frac{4-x^2}{x+7}$

Problem 15: Use Squeeze Theorem to evaluate the following limits:

(a) $\lim_{x\to 0} x\sin(1/x)$

(b) $\lim_{x\to 0} x^2\cos(1/x)$

(c) $\lim_{x\to 0} |x|\cos^2(1/x)$

(d) $\lim_{x\to 0} x^3 e^{\sin(1/x)}$

(e) $\lim_{x\to\infty} \frac{x^x}{(2x)!}$

(f) $\lim_{x\to\infty} (x!)^{1/x^2}$

Problem 16: Use the Intermediate Value Theorem to show there is a solution to the following equations over the given interval:

(a) $4^x = x^2 + 1$ over $[-2, 1]$

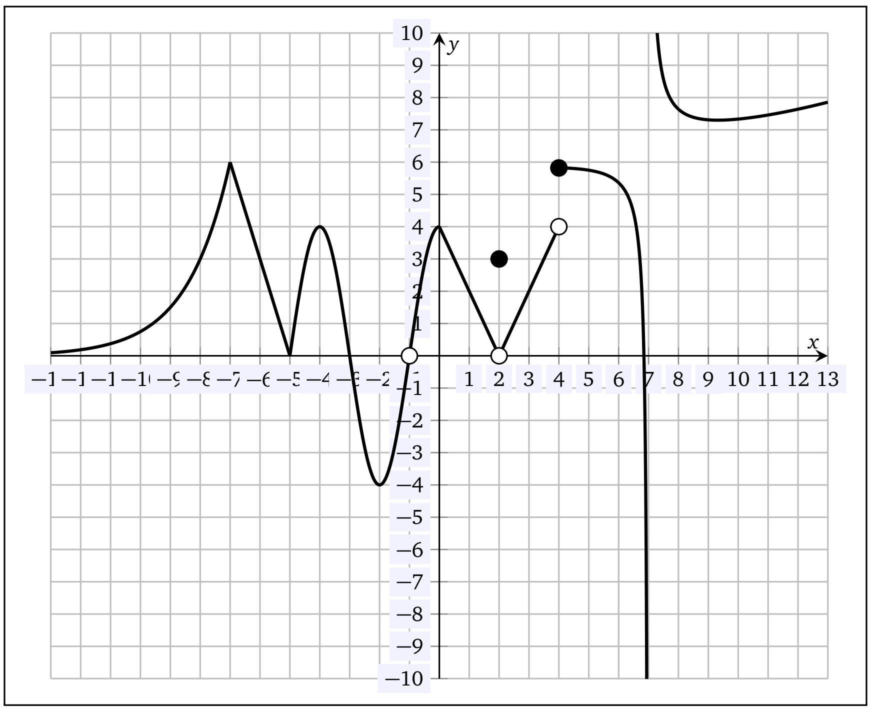
(b) $x^3 + \cos x = 2$ over $[0, 10]$

(c) $e^{-x^2} - x = 0$ over $[0, 1]$

(d) $x^3 + x + 1 = 0$ over $[-1, 0]$

(e) $\pi^{13.475} x^{15} - \sqrt{e^3} x^1 2 - x^9 + 1478x + 14.2345 = e^{\pi} x^{13} - \sqrt{1 + \sqrt{2 + \sqrt{3}}} x^{10} - 99.99x^2 + 2^{4^{6^8}}$

Problem 17: A function $f(x)$ is plotted above. Use this plot to answer the following questions:



(a) $\lim_{x \to 2^+} f(x)$

(b) $\lim_{x \to 2^-} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $f(2)$

(e) $\lim_{x \to -2^-} f(x)$

(f) $\lim_{x \to -2^+} f(x)$

(g) $\lim_{x \to -2} f(x)$

(h) $f(-2)$

(i) $\lim_{x \to 4^-} f(x)$

(j) $\lim_{x \to 4^+} f(x)$

(k) $\lim_{x \to 4} f(x)$

(l) $f(4)$

(a) What is $f(2)$?

(b) $\lim_{x \to 2^+} f(x)$

(c) $\lim_{x \to 2^-} f(x)$

(d) $\lim_{x\to 2} f(x)$

(e) $\lim_{x\to 4^-} f(x)$

(f) $\lim_{x\to 4^+} f(x)$

(g) $\lim_{x\to 4} f(x)$

(h) What is $f(4)$?

(i) $\lim_{x\to -1^-} f(x)$

(j) $\lim_{x\to -1^+} f(x)$

(k) $\lim_{x\to -1} f(x)$

(l) $\lim_{x\to -7^-} f(x)$

(m) $\lim_{x\to -7^+} f(x)$

(n) What are the zeros of $f(x)$?

(o) What is the y-intercept for $f(x)$?

(p) Where is $f(x)$ continuous?

(q) Find and classify the discontinuities of $f(x)$

(r) $\lim_{x\to \infty} f(x)$

(s) $\lim_{x\to -\infty} f(x)$

(t) Does $f(x)$ have any horizontal asymptotes?

(u) Give at least 6 points at which $f(x)$ is not differentiable. Explain why $f(x)$ is not differentiable there.