Problem 1: $\displaystyle\int_1^3 x^2 - x + 4\,dx$

Problem 2: $\displaystyle\int x\sin x^2\,dx$

Problem 3: $\displaystyle\int_0^1 \frac{dx}{x^2+1}$

Problem 4: $\displaystyle\int_{-1}^3 e^{x+1}\,dx$

Problem 5: $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Problem 6: $\displaystyle\int \frac{x^3+x^2+1}{x^2}\,dx$

Problem 7: $\displaystyle\int x^{10}+2^x\,dx$

Problem 8: $\displaystyle\int \frac{\cos x}{1+\sin x}\,dx$

Problem 9: $\displaystyle\int \frac{5x}{\sqrt{2x^2+1}}\,dx$

Problem 10: $\displaystyle\int \frac{\sqrt{x}-x^2+\sqrt[3]{x}-1}{\sqrt[3]{x}}\,dx$

Problem 11: $\displaystyle\int \frac{3x}{(x^2+3)^8}\,dx$

Problem 12: $\displaystyle\int_0^2 (x+2)^2\,dx$

Problem 13: $\displaystyle\int \frac{(\ln x)^4}{x}\,dx$

Problem 14: $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$

Problem 15: $\displaystyle\int \frac{\cos\ln x}{x}\,dx$

Problem 16: $\displaystyle\int \frac{x^4+1}{x^5+5x+2}\,dx$

Problem 17: $\displaystyle\int \frac{x-1}{\sqrt{x+1}}\,dx$

Problem 18: $\displaystyle\int_0^{\pi/2} \sin^4 x\,\cos x\,dx$

Problem 19: $\displaystyle\int \sin x\,\cos^2 x\,dx$

Problem 20: $\displaystyle\int_0^2 |2x-3|\,dx$

Problem 21: $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

Problem 22: $\displaystyle\int \frac{e^x}{e^{2x}+4}\,dx$

Problem 23: $\displaystyle\frac{d}{dx}\int_\pi^x \sqrt{t+\sqrt{\sin^2 t}}\,dt$

Problem 24: $\displaystyle\frac{d}{dx}\int_2^{1/x} \arctan t\,dt$

Problem 25: $\displaystyle\frac{d}{dx}\int_x^1 \frac{dt}{t^3+1}\,dt$

Problem 26: $\displaystyle\frac{d}{dx}\int_{2x}^4 \cos t^2\,dt$

Problem 27: $\displaystyle\frac{d}{dx}\int_x^{2x} \frac{\sin t}{t^2+t+1}\,dt$

Problem 28: $\displaystyle\frac{d}{dx}\int_{\sin^2 x}^{\ln x} e^{-t^2}\,dt$

Solutions

Solution 1:

$$\begin{aligned}\int_1^3 x^2 - x + 4\,dx &= \left(\frac{x^3}{3} - \frac{x^2}{2} + 4x\right)\Big|_1^3 \\ &= \left(\frac{3^3}{3} - \frac{3^2}{2} + 4(3)\right) - \left(\frac{1}{3} - \frac{1}{2} + 4(1)\right) \\ &= \frac{38}{3}\end{aligned}$$

Solution 2: Let $u = x^2$ so that $du = 2x\,dx$ - that is, $dx = \frac{du}{2x}$. This gives

$$\int x \sin x^2\,dx = \int \frac{\sin u}{2}\,du = \frac{1}{2}\int \sin u\,du = \frac{-\cos u}{2} + C = \frac{-\cos x^2}{2} + C$$

Solution 3: $\displaystyle\int_0^1 \frac{dx}{x^2+1} = \arctan x\Big|_0^1 = \arctan(1) - \arctan(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

Solution 4: Let $u = x + 1$ so that $du = 1\,dx$. If $x = -1$ then $u = (-1) + 1 = 0$ while if $x = 3$ then $u = 3 + 1 = 4$. This gives

$$\int_{-1}^3 e^{x+1}\,dx = \int_0^4 e^u\,du = e^u\Big|_0^4 = e^4 - e^0 = e^4 - 1$$

Solution 5: Let $u = \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}\,dx$. Therefore, $dx = 2\sqrt{x}\,du$. This gives

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u \cdot 2\,du = 2\int \sin u\,du = -2\cos u + C = -2\cos\sqrt{x} + C$$

Solution 6: $\displaystyle\int \frac{x^3 + x^2 + 1}{x^2}\,dx = \int x + 1 + \frac{1}{x^2}\,dx = \frac{x^2}{2} + x - \frac{1}{x} + C$

Solution 7: $\displaystyle\int x^{10} + 2^x\,dx = \frac{x^{11}}{11} + \frac{2^x}{\ln 2} + C$

Solution 8: Let $u = 1 + \sin x$. Then $du = \cos x\,dx$ so that $dx = \frac{du}{\cos x}$. Therefore,

$$\int \frac{\cos x}{1 + \sin x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|1 + \sin x| + C$$

Solution 9: Let $u = 2x^2 + 1$ so that $du = 4x\ dx$. Therefore, $dx = \dfrac{du}{4x}$. Then

$$\int \frac{5x}{\sqrt{2x^2+1}}\ dx = \int \frac{5}{4\sqrt{u}}\ du = \frac{5}{4}\int \frac{du}{\sqrt{u}} = \frac{5}{4}\cdot 2u^{1/2} + C = \frac{5}{2}\sqrt{2x^2+1} + C$$

Solution 10:

$$\int \frac{\sqrt{x} - x^2 + \sqrt[3]{x} - 1}{\sqrt[3]{x}}\ dx = \int x^{1/6} - x^{5/3} + 1 - x^{-1/3}\ dx = \frac{6}{7}x^{7/6} - \frac{3}{8}x^{8/3} + x - \frac{3}{2}x^{2/3} + C$$

Solution 11: Let $u = x^2 + 3$. Then $du = 2x\ dx$ so that $dx = \dfrac{du}{2x}$. Then

$$\int \frac{3x}{(x^2+3)^8}\ dx = \int \frac{3}{2u^8}\ du = \frac{3}{2}\cdot\frac{1}{-7u^7} + C = \frac{-3}{14(x^2+3)^7} + C$$

Solution 12: $\displaystyle\int_0^2 (x+2)^2\ dx = \int_0^2 x^2 + 4x + 4\ dx = \frac{x^3}{3} + 2x^2 + 4x\Big|_0^2 = \frac{56}{3} - 0 = \frac{56}{3}$

Solution 13: Let $u = \ln x$ so that $du = \dfrac{dx}{x}$. Therefore, $dx = x\ du$. Then we have

$$\int \frac{(\ln x)^4}{x}\ dx = \int u^4\ du = \frac{u^5}{5} + C = \frac{(\ln x)^5}{5} + C$$

Solution 14: Let $u = \dfrac{1}{x}$ so that $du = \dfrac{-1}{x^2}\ dx$. Then $dx = -x^2\ du$. Therefore,

$$\int \frac{e^{1/x}}{x^2}\ dx = \int -e^u\ du = -e^u + C = -e^{1/x} + C$$

Solution 15: Let $u = \ln x$ so that $du = \dfrac{1}{x}\ dx$. Therefore, $dx = x\ du$ so that

$$\int \frac{\cos \ln x}{x}\ dx = \int \cos u\ du = \sin u + C = \sin \ln x + C$$

Solution 16: Let $u = x^5 + 5x + 2$. Then $du = 5x^4 + 5\ dx = 5(x^4+1)\ dx$ so that $dx = \dfrac{du}{5(x^4+1)}$.
Therefore,

$$\int \frac{x^4+1}{x^5+5x+2}\ dx = \int \frac{1}{5u}\ du = \frac{1}{5}\int \frac{du}{u} = \frac{1}{5}\ln|u| + C = \frac{\ln|x^5+5x+2|}{5} + C$$

Solution 17: Let $u = x+1$ so that $du = dx$. Notice that $u = x+1$ so that $x = u-1$. But then $x-1 = (u-1)-1 = u-2$. Therefore,

$$\int \frac{x-1}{\sqrt{x+1}}\,dx = \int \frac{u-2}{\sqrt{u}}\,du = \int \sqrt{u} - \frac{2}{\sqrt{u}}\,du = \frac{2}{3}u^{3/2} - 4\sqrt{u} + C = \frac{2}{3}\sqrt{(x+1)^3} - 4\sqrt{x+1} + C$$

Solution 18: Let $u = \sin x$ so that $du = \cos x\,dx$. But then $dx = \dfrac{du}{\cos x}$. Therefore,

$$\int \sin^4 x\,\cos x\,dx = \int u^4\,du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$$

Solution 19: Let $u = \cos x$ so that $du = -\sin x\,dx$. Therefore, $dx = \dfrac{du}{-\sin x}$. Then

$$\int \sin x\,\cos x^2\,dx = \int -u^2\,du = \frac{-u^3}{3} + C = \frac{-\cos^3 x}{3} + C$$

Solution 20:

$$\begin{aligned}\int_0^2 |2x-3|\,dx &= \int_0^{3/2} |2x-3|\,dx + \int_{3/2}^2 |2x-3|\,dx \\ &= \int_0^{3/2} -(2x-3)\,dx + \int_{3/2}^2 2x-3\,dx \\ &= (3x-x^2)\Big|_0^{3/2} + (x^2-3x)\Big|_{3/2}^2 \\ &= \left(\frac{9}{4}-0\right) + \left(-2-\frac{-9}{4}\right) \\ &= \frac{5}{2}\end{aligned}$$

Solution 21: Let $u = e^x + 1$ so that $du = e^x\,dx$. Then $dx = \dfrac{du}{e^x}$. But then

$$\int \frac{e^x}{e^x+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|e^x+1| + C$$

Solution 22: Let $u = e^x$ so that $du = e^x\,dx$. Then $dx = \dfrac{du}{e^x}$. But then

$$\int \frac{e^x}{e^{2x}+4}\,dx = \int \frac{e^x}{(e^x)^2+4}\,dx = \int \frac{du}{u^2+4} = \int \frac{du}{u^2+4}\cdot\frac{1/4}{1/4} = \int \frac{1/4}{\frac{u^2}{4}+1}\,du = \frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2+1}$$

Now let $w = \frac{u}{2}$ so that $dw = \frac{du}{2}$. But then $du = 2\,dw$. Therefore,

$$\frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2 + 1} = \frac{1}{4}\int \frac{2\,dw}{w^2+1} = \frac{1}{2}\int \frac{dw}{1+w^2} = \frac{1}{2}\arctan w + C = \frac{1}{2}\arctan\left(\frac{u}{2}\right) + C = \frac{1}{2}\arctan\left(\frac{e^x}{2}\right) + C$$

Solution 23:

$$\frac{d}{dx}\int_{\pi}^{x} \sqrt{t + \sqrt{\sin^2 t}}\,dt = \sqrt{x + \sqrt{\sin^2 x}}$$

Solution 24:

$$\frac{d}{dx}\int_{2}^{1/x} \arctan t\;dt = \arctan\left(\frac{1}{x}\right)\cdot\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-\arctan(1/x)}{x^2}$$

Solution 25:

$$\frac{d}{dx}\int_{x}^{1} \frac{dt}{t^3+1}\;dt = -\frac{d}{dx}\int_{1}^{x}\frac{dt}{t^3+1} = \frac{-1}{x^3+1}$$

Solution 26:

$$\frac{d}{dx}\int_{2x}^{4} \cos t^2\;dt = -\frac{d}{dx}\int_{4}^{2x} \cos t^2\;dt = -\cos((2x)^2)\cdot\frac{d}{dx}(2x) = -2\cos(4x^2)$$

Solution 27:

$$\begin{aligned}
\frac{d}{dx}\int_{x}^{2x} \frac{\sin t}{t^2+t+1}\;dt &= \frac{d}{dx}\left(\int_{x}^{0}\frac{\sin t}{t^2+t+1}\;dt + \int_{0}^{2x}\frac{\sin t}{t^2+t+1}\;dt\right) \\
&= \frac{d}{dx}\left(-\int_{0}^{x}\frac{\sin t}{t^2+t+1}\;dt + \int_{0}^{2x}\frac{\sin t}{t^2+t+1}\;dt\right) \\
&= -\frac{d}{dx}\int_{0}^{x}\frac{\sin t}{t^2+t+1}\;dt + \frac{d}{dx}\int_{0}^{2x}\frac{\sin t}{t^2+t+1}\;dt \\
&= -\frac{\sin x}{x^2+x+1} + \frac{\sin(2x)}{(2x)^2+(2x)+1}\cdot\frac{d}{dx}(2x) \\
&= -\frac{\sin x}{x^2+x+1} + \frac{2\sin(2x)}{4x^2+2x+1}
\end{aligned}$$

Solution 28:

$$\begin{aligned}
\frac{d}{dx}\int_{\sin^2 x}^{\ln x} e^{-t^2}\,dt &= \frac{d}{dx}\left(\int_{\sin^2 x}^{0} e^{-t^2}\,dt + \int_{0}^{\ln x} e^{-t^2}\,dt\right)\\
&= \frac{d}{dx}\left(-\int_{0}^{\sin^2 x} e^{-t^2}\,dt + \int_{0}^{\ln x} e^{-t^2}\,dt\right)\\
&= -\frac{d}{dx}\int_{0}^{\sin^2 x} e^{-t^2}\,dt + \frac{d}{dx}\int_{0}^{\ln x} e^{-t^2}\,dt\\
&= -e^{-(\sin^2 x)^2}\cdot\frac{d}{dx}(\sin^2 x) + e^{-(\ln x)^2}\cdot\frac{d}{dx}(\ln x)\\
&= -2e^{-\sin^4 x}\sin x\cos x + \frac{e^{-(\ln x)^2}}{x}
\end{aligned}$$