Name: _____ **Quiz 10**

MAT 295 **Fall 2016**

Problem 1: Verify that the function $f(x) = \dfrac{x-1}{x+2}$ satisfies the condition for the Mean Value Theorem on $[0,2]$ and find any points $c \in [0,2]$ satisfying the condition guaranteed by the theorem.

Problem 2: Find $\lim\limits_{n\to\infty} \left(\cos\dfrac{1}{n}\right)^{2n^2}$.

Problem 3: Assuming a classical model of the atom, Niels Bohr was able to show that the energy of a hydrogen atom with separation r between the proton and the election is given by

$$E(r) = \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

where $\hbar$ is the reduced Planck's constant (Dirac constant), m_e is the mass of the election, e is the charge of an electron, and ϵ_0 is permittivity of free space. The Bohr radius for the hydrogen atom, denoted r_{Bohr}, is the radius at which $E(r)$ is minimal and it is approximately the expected distance between the proton and the electron in the ground state. Show that

$$r_{\text{Bohr}} = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$$