Name: *Caleb McWhorter — Solutions* **Quiz 12**
MAT 295 **Fall 2016**

Problem 1: Using 4 subintervals, approximate the integral $\int_0^4 x^2\,dx$ using the following methods:

(a) Left Hand Sums

$$1\Big(f(0)+f(1)+f(2)+f(3)\Big)=1(0+1+4+9)=14$$

(b) Right Hand Sums

$$1\Big(f(1)+\cdot f(2)+\cdot f(3)+\cdot f(4)\Big)=1(1+4+9+16)=30$$

(c) Midpoint Rule

$$1\Big(f(0.5)+f(1.5)+f(2.5)+f(3.5)\Big)=0.25+2.25+6.25+12.25=21$$

(d) Trapezoidal Rule

$$\frac{1}{2}\Big(f(0)+2f(1)+2f(2)+2f(3)+f(4)\Big)=\frac{1}{2}\Big(0+2+8+18+16)=\frac{\textit{Left Sum}+\textit{Right Sum}}{2}=22$$

(e) Simpson's Rule

$$\frac{1}{3}\Big(f(0)+4\cdot f(1)+2f(2)+4f(3)+f(4)\Big)=\frac{1}{3}\Big(4+8+36+16\Big)=\frac{2\cdot\textit{Mid Sum}+\textit{Trap Sum}}{3}=\frac{64}{3}\approx 21.3$$

Note: With 4 subintervals, $\Delta x=\dfrac{4-0}{4}=1$,

x	0	1	2	3	4
$f(x)$	0	1	4	9	16

, and $\displaystyle\int_0^4 x^2\,dx=\frac{64}{3}$.

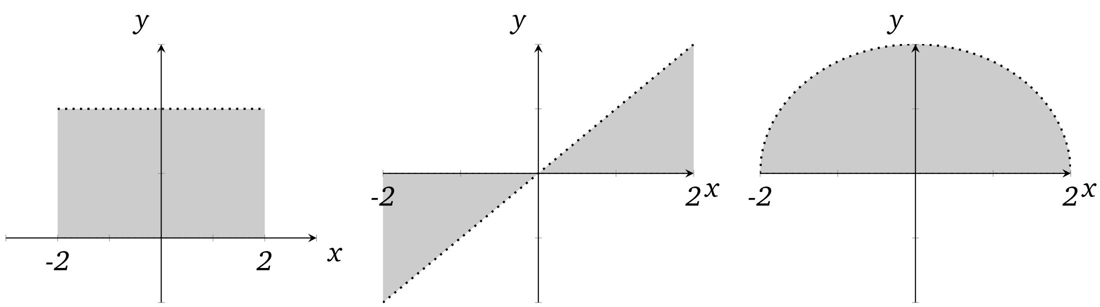
Problem 2: Evaluate the following sum by representing it as a Riemann integral and evaluating the integral:

$$\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{2}{n}\left(\left(\frac{2i}{n}\right)^3+\frac{6i}{n}-10\right)$$

$$\begin{aligned}
\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{2}{n}\left(\left(\frac{2i}{n}\right)^3+\frac{6i}{n}-10\right) &= \lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{2-0}{n}\left(\left(2\cdot\frac{i}{n}\right)^3+3\left(2\cdot\frac{i}{n}\right)-10\right)\\
&=\int_0^2 (x^3+3x-10)\,dx\\
&=\left(\frac{x^4}{4}+\frac{3x^2}{2}-10x\right)\Bigg|_0^2\\
&=\left(\frac{16}{4}+\frac{3(4)}{2}-10(2)\right)-0\\
&=-10
\end{aligned}$$

Problem 3: Evaluate the integral by interpreting it in terms of area: $\int_{-2}^{2}\left(1+x-\sqrt{4-x^2}\right)\,dx$

$$\int_{-2}^{2}\left(1+x-\sqrt{4-x^2}\right)\,dx=\int_{-2}^{2}1\,dx+\int_{-2}^{2}x\,dx-\int_{-2}^{2}\sqrt{4-x^2}\,dx$$



The regions for the integration are shown above. The first has area $A=\ell w=1(4)=4$. For the second, the areas of the triangles cancel (one above and one below the axis). For the third, we have area $A=\pi r^2\cdot\frac{1}{2}=\pi(2^2)\cdot\frac{1}{2}=2\pi$—the half coming from the fact it is a half circle. Then we have

$$\int_{-2}^{2}\left(1+x-\sqrt{4-x^2}\right)\,dx=4+0-2\pi=4-2\pi=2(2-\pi)$$