Name: *Caleb McWhorter — Solutions* **Quiz 13**
MAT 295 **Fall 2016**

Problem 1: Suppose $\int_0^{10} f(x)\,dx = 8$, $\displaystyle\int_0^3 f(x)\,dx = -2$, and $\displaystyle\int_3^{10} g(x) = 3$.

What is $\displaystyle\int_3^{10} \big(f(x) + 2g(x)\big)\,dx$?

$$8 = \int_0^{10} f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^{10} f(x)\,dx = -2 + \int_3^{10} f(x)\,dx = 10 + 2(3) = 16$$

But then $\displaystyle\int_3^{10} f(x)\,dx = 10$. *But then*

$$\int_3^{10} \big(f(x) + 2g(x)\big)\,dx = \int_3^{10} f(x)\,dx + 2\int_3^{10} g(x)\,dx = 10 + 2(3) = 16$$

Problem 2: Evaluate the following integrals:

$$\int \frac{\sin x}{\cos^4 x}\,dx$$

Let $u = \cos x$. *Then* $du = -\sin x\,dx$ *so that* $dx = \dfrac{du}{-\sin x}$. *Then*

$$\int \frac{\sin x}{\cos^4 x}\,dx = -\int \frac{du}{u^4} = \frac{1}{3u^3} + C = \frac{1}{3\cos^3 x} + C = \frac{1}{3}\sec^3 x + C$$

$$\int x^2 \cos x^3\,dx =$$

Let $u = x^3$. *Then* $du = 3x^2\,dx$ *so that* $dx = \dfrac{du}{3x^2}$. *Then*

$$\int x^2 \cos x^3\,dx = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin x^3 + C$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx =$$

Let $u = \sqrt{x}$. *Then* $du = \dfrac{1}{2\sqrt{x}}\,dx$ *so that* $dx = 2\sqrt{x}\,du$. *Then*

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$$

Problem 3: Find $\frac{d}{dx}\int_x^{2x} \frac{dt}{\sqrt{1+t^2}}$

$$\begin{aligned}
\frac{d}{dx}\int_x^{2x} \frac{dt}{\sqrt{1+t^2}} &= \frac{d}{dx}\left(\int_x^{0} \frac{dt}{\sqrt{1+t^2}} + \int_0^{2x} \frac{dt}{\sqrt{1+t^2}}\right) \\
&= \frac{d}{dx}\left(-\int_0^{x} \frac{dt}{\sqrt{1+t^2}} + \int_0^{2x} \frac{dt}{\sqrt{1+t^2}}\right) \\
&= \frac{-1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+(2x)^2}}\cdot 2
\end{aligned}$$

Problem 4: Find $\frac{d}{dx}\left(\int_{x^2}^{0} \cos t\ dt\right)^3$

$$\begin{aligned}
\frac{d}{dx}\left(\int_{x^2}^{0} \cos t\ dt\right)^3 &= 3\left(\int_{x^2}^{0} \cos t\ dt\right)^2 \cdot \frac{d}{dx}\int_{x^2}^{0} \cos t\ dt \\
&= 3\left(\sin t\Big|_{x^2}^{0}\right)\cdot(-\cos x^2)\cdot 2x \\
&= 3\big(\sin 0 - \sin x^2\big)^2\cdot(-\cos x^2)\cdot 2x \\
&= -6x\sin^2(x^2)\cos x^2
\end{aligned}$$

Problem 5: Find the average value of $\sin x$ on $[0, \pi]$.

$$\frac{1}{\pi - 0}\int_0^{\pi} \sin x\ dx = \frac{1}{\pi}\cdot -\cos x\Big|_0^{\pi} = \frac{1}{\pi}(-(-1)-(-1)) = \frac{2}{\pi}$$