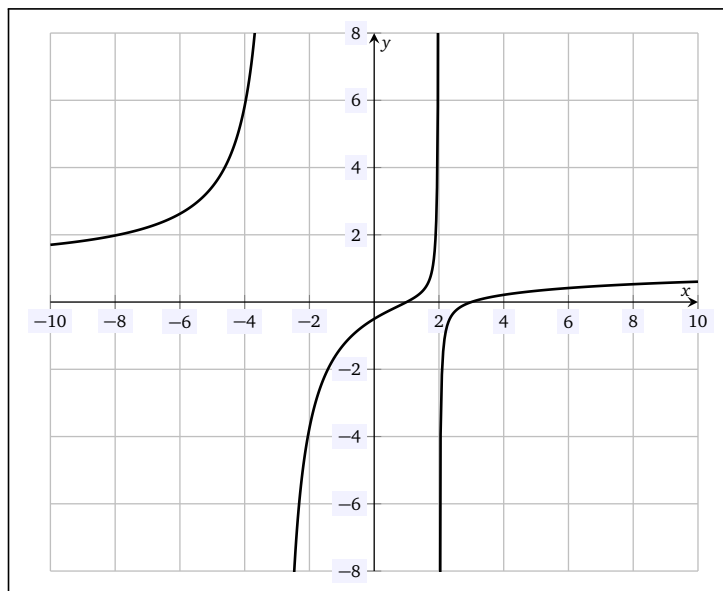


**Problem 1:** The plot of a function  $f(x)$  is given below. Use the plot to evaluate the following limits:



(a)  $\lim_{x \rightarrow 2^+} f(x) = -\infty$

(f)  $\lim_{x \rightarrow -3} f(x) = DNE$

(b)  $\lim_{x \rightarrow 2^-} f(x) = \infty$

(g)  $\lim_{x \rightarrow \infty} f(x) = 1$

(c)  $\lim_{x \rightarrow -3^-} f(x) = \infty$

(h)  $\lim_{x \rightarrow -\infty} f(x) = 1$

(d)  $\lim_{x \rightarrow -3^+} f(x) = -\infty$

(e)  $\lim_{x \rightarrow 2} f(x) = DNE$

(i) What are the roots of  $f(x)$ ?  $x = 1, 3$

**Problem 2:** Evaluate the following limits. You do not need to justify your answer.

(a)  $\lim_{x \rightarrow \infty} \frac{2x^2 - 5x + 7}{7x^3 - 2x^2 + 6} = 0$

(e)  $\lim_{x \rightarrow \infty} \frac{\sin x^2}{5x} = 0$

(b)  $\lim_{x \rightarrow \infty} \frac{2x^5 + 4x^2 + 7}{3x^5 - 4x^3 + 4x + 1} = 2/3$

(f)  $\lim_{x \rightarrow \infty} \frac{x^2 + 7x + 3}{\sqrt{x-3}} = \infty$

(c)  $\lim_{x \rightarrow \infty} \frac{2^x}{x^3 + 2x + 1} = \infty$

(g)  $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = 0$

(d)  $\lim_{x \rightarrow \infty} \frac{5 \ln x}{x^2 + 2x + 3} = 0$

(h)  $\lim_{x \rightarrow \infty} \frac{x^3 + 5x + 9}{\sqrt{x^{10} - 4x + 6}} = 0$

**Problem 3:**

$$f(x) = \frac{(x+3)(x-2)(x+6)}{(x-2)(x+1)(x-3)}$$

(a) What are the  $x$ -intercepts for  $f(x)$ ?

$$x = -3, -6$$

(b) What is the  $y$ -intercept for  $f(x)$ ?

$$y = f(0) = \frac{18}{-3} = -6$$

(c) Where is  $f(x)$  continuous?

*Everywhere on  $\mathbb{R}$  except for  $x = 2, -1, 3$*

(d) What are vertical asymptotes for  $f(x)$ ?

$$x = -1, 3$$

(e) What are the horizontal asymptotes for  $f(x)$ ?

$$y = \lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow \infty} f(x) = 1$$

(f) Does  $f(x)$  have any removable discontinuities? If so, what is the point?

$$f(x) = \frac{(x+3)(x+6)}{(x+1)(x-3)}; \quad f(2) = \frac{-40}{3}$$

*Then  $f(x)$  has a removable discontinuity (or hole) at  $(2, -40/3)$ .*

**Problem 4:** Evaluate the following limit. Be sure to justify your answer completely:  $\lim_{x \rightarrow -\infty} \frac{x^3 + 2x + 3}{x^2 + 6x + 1}$

$$\lim_{x \rightarrow -\infty} \frac{x^3 + 2x + 3}{x^2 + 6x + 1} = \lim_{x \rightarrow -\infty} \frac{x + \frac{2}{x} + \frac{3}{x^2}}{1 + \frac{6}{x} + \frac{1}{x^2}} = -\infty$$