Name: *Caleb McWhorter — Solutions* **Quiz 4**

MAT 295 **Fall 2016**

Problem 1: Determine the derivative of the following:

(a) $\dfrac{d}{dx}\arctan x = \dfrac{1}{1+x^2}$

(b) $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt[7]{x^3}}\right) = \dfrac{-3}{7\sqrt[4]{x^{10}}}$

(c) $\dfrac{d}{dx}\tan x = \sec^2 x$

(d) $\dfrac{d}{dx}\dfrac{1}{5^x} = \dfrac{-\ln 5}{5^x}$

(e) $\dfrac{d}{dx}\sec x = \sec x \tan x$

(f) $\dfrac{d}{dx}\log_7 x = \dfrac{1}{x\ln 7}$

(g) $\dfrac{d}{dx}e^x = e^x$

(h) $\dfrac{d}{dx}\cot x = -\csc^2 x$

(i) $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$

(j) $\dfrac{d}{dx}(2x^3 - x^2 + 4x - 7) = 6x^2 - 2x + 4$

(k) $\dfrac{d}{dx}\cos^{-1} x = \dfrac{-1}{\sqrt{1-x^2}}$

Problem 2: Use a tangent line to $f(x) = \sqrt{x}$ to approximate $\sqrt{26}$. Check how "close" your answer is to the actual value by computing its square. Determine whether your answer is an overestimation or an underestimation two ways: first by using the square of your approximation and second by graphing $f(x)$ and its tangent line.

Solution. *The equation for a tangent line for $f(x)$ at $x = a$ is...*

$$l_a(x) = f(a) + f'(a)(x-a)$$

We know $\sqrt{25} = 5$, so we will use the tangent line at $x = 25$ for $f(x) = \sqrt{x}$ to approximate $\sqrt{26}$.

$$f(x) = \sqrt{x} \Rightarrow f(5) = \sqrt{25} = 5$$
$$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$

We should have $\sqrt{26} \approx l_{25}(26) = 5 + \frac{1}{10}(26-25) = 5 + \frac{1}{10} = 5.10$. Observe that $(5.10)^2 = (5+0.10)(5+0.10) = 25 + 0.5 + 0.5 + 0.01 = 26.01$. So our approximation is 'close,' Because $5.10^2 > 26$, we have over-approximated. We can also see this from either the derivative (why?) and the graph of $f(x)$