Name: *Caleb McWhorter — Solutions* **Quiz 8**

MAT 295 **Fall 2016**

Problem 1: Use l'Hôpital's Rule to evaluate the following limits:

$$\lim_{x\to 1} \frac{\sqrt{2-x}-x}{x-1} \overset{L.H.}{=} \lim_{x\to 1} \frac{\frac{1}{2}\cdot\frac{-1}{\sqrt{2-x}}-1}{1} = \lim_{x\to 1}\left(\frac{-1}{2\sqrt{2-x}}-1\right) = \frac{-1}{2}-1 = -\frac{3}{2}$$

$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{x}{x\sin x}-\frac{\sin x}{x\sin x}\right) = \lim_{x\to 0}\frac{x-\sin x}{x\sin x} \overset{L.H.}{=} \lim_{x\to 0}\frac{1-\cos x}{\sin x + x\cos x} \overset{L.H.}{=} \lim_{x\to 0}\frac{\sin x}{\cos x + (\cos x - x\sin x)} = 0$$

$$\lim_{x\to 0^+}\frac{\cot x}{\ln x} \overset{L.H.}{=} \lim_{x\to 0^+}\frac{-\csc^2 x}{1/x} = \lim_{x\to 0^+}\frac{-1}{2\sin x\cos x} = -\infty$$

$$\lim_{x\to 0^+}(\sin x)^{\tan x}$$

Let $y = (\sin x)^{\tan x}$. *Then* $\ln y = \ln(\sin x)^{\tan x} = \tan x \ln\sin x$. *Then*

$$\lim_{x\to 0^+}\ln y = \lim_{x\to 0^+}\tan x\ln\sin x = \lim_{x\to 0^+}\frac{\ln\sin x}{\cot x} \overset{L.H.}{=} \lim_{x\to 0^+}\frac{\frac{\cos x}{\sin x}}{-\csc^2 x} = \lim_{x\to 0^+}\frac{\cos x}{\sin x}\cdot(-\sin^2 x) = \lim_{x\to 0^+}\cos x\cdot(-\sin x) = 0$$

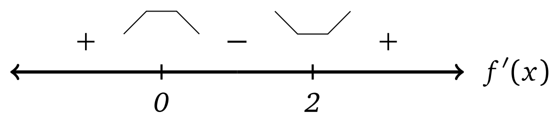
But then $\lim_{x\to 0^+}\ln y = 0$ *so that* $\lim_{x\to 0^+} y = e^0 = 1$. *Therefore,* $\lim_{x\to 0^+}(\sin x)^{\tan x} = 1$.

Problem 2: For the function $f(x) = x^3 - 3x^2 + 5$, find the local and absolute max/mins on the interval $[-2, 3]$. Use the First Derivative Test to determine any local max/mins and check using the Second Derivative Test.

We know that

$$f(x) = x^3 - 3x^2 + 5$$
$$f'(x) = 3x^2 - 6x = 3x(x-2)$$
$$f''(x) = 6x - 6 = 6(x-1)$$

Now $f'(x)$ is never undefined so the only critical values are when $f'(x) = 0$. Now $f'(x) = 0$ means that $3x(x-2) = 0$. But then $x = 0$ or $x = 2$. Using the First Derivative Test,



Therefore, the local min is at $x = 2$ and the local max is at $x = 0$. We know that $f(0) = 0 - 0 + 5 = 5$ and $f(2) = 2^3 - 3(2^2) + 5 = 8 - 12 + 5 = 1$. Then $(0, 5)$ is a local max and $(2, 1)$ is a local min. We check using the Second Derivative Test.

$$f''(0) = 6(0-1) = -6 < 0$$
$$f''(2) = 6(2-1) = 6 > 0$$

As $f''(0) < 0$, we know that there is a local max at $x = 0$ and as $f''(2) > 0$, we know there is a local minimum at $x = 2$. Finally,

$$f(-2) = (-2)^3 - 3(-2)^2 + 5 = -15$$
$$f(3) = 3^3 - 3 \cdot 3^2 + 5 = 5$$

Then $(2, 1)$ is a local minimum on $[-2, 3]$ while $(0, 5)$ is a local max on $[-2, 3]$. The absolute maximum is 5 at $x = 0$ and $x = 3$ and the absolute minimum is -15 at $x = -2$.