Math 397 Spring 2016 Exam I

Name: _____
(Please Print.)

Do all your work on this exam. Correct answers should be supported by
your calculations and reasoning where appropriate.

1. (a) Find all unit vectors parallel to $\bar{v} = \langle 1, -3, \sqrt{2} \rangle$.

Problem	Points	Score
1	8	
2	8	
3	10	
4	8	
5	10	
6	10	
Total	60	

(b) Which, if any, of the following vectors is orthogonal to $\bar{v} = \langle 2, -3, \sqrt{2} \rangle$? (Show your computations.)

$\bar{a} = \langle 3, 2, 5\sqrt{2} \rangle$, $\bar{b} = \langle -6, 2, 6\sqrt{2} \rangle$

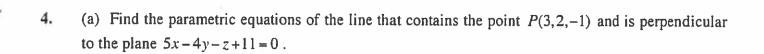(c) Assume $\bar{u} \bullet \bar{v} = 8$ and $\bar{u} \bullet \bar{w} = -5$ find

(i) $\bar{u} \bullet (3\bar{v} + 2\bar{w})$

(ii) For what value of k is $\bar{v} + k\bar{w}$ orthogonal to $\bar{u}$?

2.   (a) Find the point of intersection of the following pair of lines.

$$x = 1 + t \qquad\qquad\qquad\qquad x = 4 - s$$

$$L_1: \; y = 2 + 2t \quad -\infty < t < \infty \quad \text{and} \quad L_2: \; y = 2 + s \quad -\infty < s < \infty$$

$$z = 4 - t \qquad\qquad\qquad\qquad z = 2s - 1$$

(b)   Find the angle between the lines in part (a).

$$x = 1 + t \qquad\qquad\qquad\qquad x = 4 - s$$

$$L_1: \; y = 2 + 2t \quad -\infty < t < \infty \quad \text{and} \quad L_2: \; y = 2 + s \quad -\infty < s < \infty$$

$$z = 4 - t \qquad\qquad\qquad\qquad z = 2s - 1$$

3. (a) Find the area of the triangle in three-space with vertices $P(2,3,-1)$, $Q(4,5,2)$ and $R(6,2,1)$

(b) Find the equation of the plane through the three points in part (a)

4. (a) Find the parametric equations of the line that contains the point $P(3,2,-1)$ and is perpendicular to the plane $5x-4y-z+11=0$.

(b) Find the perpendicular distance between the point P and the plane in part (a).

5. Consider the two parallel planes $x+2y+3z=12$ containing the point $P(2,2,2)$ and $x+2y+3z=14$ containing the point $Q(5,3,1)$.

(i) Compute and <u>simplify</u> the vector projection of $\vec{PQ}$ onto the normal vector $\bar{i}+2\bar{j}+3\bar{k}=\langle 1,2,3 \rangle$.

(ii) Describe geometrically what the length of the vector you computed in part (a) gives. [A diagram may help you decide.]

6. (a) Find the center and radius of the sphere with equation $x^2 - 2x + y^2 + 6y + z^2 + 4z = 2$.

(b) Give the equation of the intersection of the sphere in part (a) with the xy-plane.