Math 397 Spring 2016 **Exam I** Name: _____

Do all your work on this exam. Correct answers should be supported by your calculations and reasoning where appropriate. Please **silence** and **put away** all cell phones and similar devices, including earbuds. **No calculators are allowed**. You need not simplify.

Problem	Points	Score
1	10	
2	12	
3	10	
4	14	
5	14	
Total	60	

1. (10 points) (a) Find the center and radius of the sphere with equation $x^2 - 2x + y^2 + 6y + z^2 + 8z + 17 = 0$.

(b) Find the intersection of the sphere in part (a) with the yz-plane.

(c) Find the intersection of the sphere in part (a) with the xz-plane. Explain how you could find this answer just by looking at the equation of the sphere.

2. (12 points) (a) Find the point of intersection of the following pair of lines.

$$L_1: \begin{aligned} x &= 3+t \\ y &= 6+2t \\ z &= 2-t \end{aligned} \quad -\infty < t < \infty \text{ and } \quad L_2: \begin{aligned} x &= -4+2s \\ 7 &= 7-s \\ z &= 12-3s \end{aligned} \quad -\infty < s < \infty$$

(b) Find the angle between the lines in part (a).

3. (10 points) (a) Given any two vectors $\mathbf{a}$ and $\mathbf{b}$, **explain** why $\mathbf{b} \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$

(b) Find parametric equations for the line containing the point (3,–6, 4) which is perpendicular to the plane $2x - 3y + 7z = 10$.

4. (14 points) (a) Find the area of the triangle with vertices at the three points $A(1, 3, -2)$, $B(-2, 3, 1)$ and $C(2, 4, 0)$.

(b) Find the equation of the plane containing the three points in part (a).

5. (14 points) Consider two parallel planes $x+2y+3z=12$ and $x+2y+3z=14$. The first plane contains the point $P(2, 2, 2)$ while the second plane contains the point $Q(5, 3, 1)$.

(a) Find the vector projection of the vector $\mathbf{PQ}$ onto the normal vector $\mathbf{n}=\langle 1, 2, 3\rangle$.

(b) Find $comp_{\mathbf{n}}\mathbf{PQ}$, the scalar projection of PQ onto n.

(c) What does your answer to part (b) tell you geometrically? (Hint: Draw a picture and try to visualize situation.)