Solutions

Exam 1 **MAT 397 Section 001** **Spring 2016**

This exam has 6 problems on 6 pages. No notes, calculators, or electronic devices of any kind are allowed. **Show all your work!**

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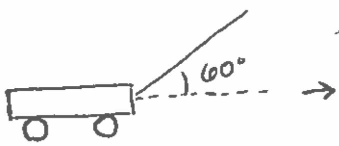
1. (10 points) A wagon is pulled along a level path for a distance of 22m by a rope held at an angle of 60° above the horizontal. If the force applied to the rope is 100N, what is the work done by the force?



$\vec{F}$ $\vec{D}$

$$W = \vec{F} \cdot \vec{D}$$
$$= |\vec{F}||\vec{D}| \cos\theta$$
$$= 100 \cdot 22 \cdot \cos 60°$$
$$= 100 \cdot 22 \cdot \tfrac{1}{2}$$
$$= 100 \cdot 11$$
$$= 1100\,J$$

2. (20 points) Given vectors $\mathbf{a} = \langle 1,2,3 \rangle$ and $\mathbf{b} = \langle 1,7,9 \rangle$,

a) find $2\mathbf{a} - \mathbf{b}$.

$$2\vec{a} - \vec{b} = 2\langle 1,2,3 \rangle - \langle 1,7,9 \rangle = \langle 2,4,6 \rangle - \langle 1,7,9 \rangle = \langle 1,-3,-3 \rangle$$

b) find $\mathbf{a} \cdot \mathbf{b}$.

$$\vec{a} \cdot \vec{b} = 1(1) + 2(7) + 3(9) = 1 + 14 + 27 = 42$$

c) find $\mathbf{a} \times \mathbf{b}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 7 & 9 \end{vmatrix}$$

$$= \hat{i}(18 - 21) - \hat{j}(9 - 3) + \hat{k}(7 - 2)$$

$$= -3\hat{i} - 6\hat{j} + 5\hat{k}$$

$$= \langle -3, -6, 5 \rangle$$

d) find the vector projection $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$ of $\mathbf{b}$ onto $\mathbf{a}$.

$$\mathrm{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a} = \frac{42}{14} \langle 1,2,3 \rangle$$

$$= 3\langle 1,2,3 \rangle$$

$$= \langle 3,6,9 \rangle$$

$$\vec{a} \cdot \vec{a} = 1^2 + 2^2 + 3^2$$

$$= 1 + 4 + 9$$

$$= 14$$

3. (20 points)

a) Find symmetric equations for the line through the points $A(5, -2, 3)$ and $B(6, -1, 5)$.

$\vec{AB} = \langle 6-5, -1-{-2}, 5-3 \rangle$ Could go from $\vec{AB}$ to solution directly...

$= \langle 1, 1, 2 \rangle$

$r(t) = \langle 1, 1, 2 \rangle t + \langle 5, -2, 3 \rangle$

$= \langle t+5,\ t-2,\ 2t+3 \rangle$

$\left\{\begin{array}{l} x = t+5 \\ y = t-2 \\ z = 2t+3 \end{array}\right.$

$\frac{x-5}{1} = \frac{y+2}{1} = \frac{z-3}{2}$

b) Find the point at which the line from part a) intersects the plane $x + y + z = 2$.

Have by above...

$\left\{\begin{array}{l} x = t+5 \\ y = t-2 \\ z = 2t+3 \end{array}\right.$

$x + y + z = 2$

$(t+5) + (t-2) + (2t+3) = 2$

$4t + 6 = 2$

$4t = -4$

$t = -1$

$x = t+5 = 4$

$y = t-2 = -3$

$z = 2t+3 = 1$

$(4, -3, 1)$

4. (20 points) Let **a** = ⟨1,2,3⟩ and **b** = ⟨1,7,9⟩ as in Problem 2, and let C be the point $C(5,1,0)$.

a) Find a scalar equation for the plane through the point C with normal vector **a**.

$$\vec{n} \cdot \vec{rP} = 0$$
$$\langle 1,2,3 \rangle \cdot \langle x-5, y-1, z-0 \rangle = 0$$
$$1(x-5) + 2(y-1) + 3(z-0) = 0$$
$$x + 2y + 3z = 7$$

b) Find a scalar equation for the plane through the point C parallel to the vectors **a** and **b**.

From 2c, we have...

$$\vec{a} \times \vec{b} = \langle -3, -6, 5 \rangle$$

Then...

$$\vec{n} \cdot \vec{rP} = 0$$
$$\langle -3, -6, 5 \rangle \cdot \langle x-5, y-1, z-0 \rangle = 0$$
$$-3(x-5) - 6(y-1) + 5(z-0) = 0$$
$$-3x + 15 - 6y + 6 + 5z = 0$$
$$3x + 6y - 5z = 15 + 6$$
$$3x + 6y - 5z = 21$$

5. (10 points) Match the graphs below to the equations in the right-hand column by putting the number of the appropriate equation into the blank beside the letter corresponding to the graph.

A 5

B 4

C 1

D 6

(1) $2z = y^2 - x^2$

(2) $x^2 + 2y^2 + z^2 = 1$

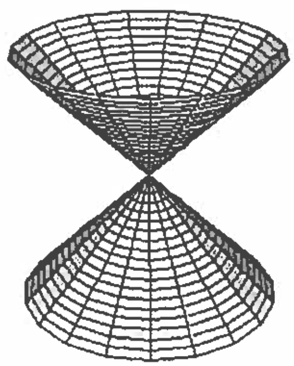
(3) $x + y - z = 1$

(4) $x^2 + y^2 - z^2 = 1$

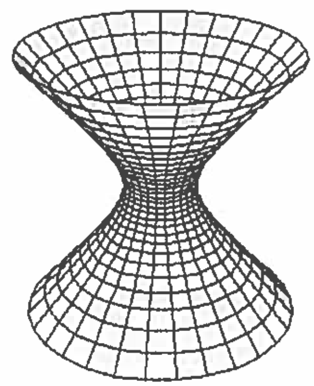
(5) $x^2 + y^2 - z^2 = 0$

(6) $x^2 + y^2 - z = 0$

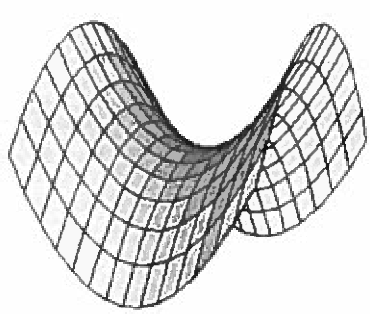
A



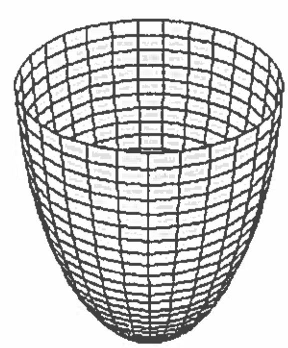
B



C



D



6. (20 points) Let $\mathbf{r}(t) = \langle \cos t\,, t \sin t\,, e^t \rangle$. Find the following.

a) The derivative $\mathbf{r}'(t)$.

$$\vec{r}\,'(t) = \langle -\sin t,\ \sin t + t\cos t,\ e^t \rangle$$

b) The unit tangent vector $\mathbf{T}(t)$. (Don't simplify your result).

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$$

$$= \frac{\langle -\sin t,\ \sin t + t\cos t,\ e^t \rangle}{\sqrt{\sin^2 t + (\sin t + t\cos t)^2 + e^{2t}}}$$

c) The unit tangent vector $\mathbf{T}(0)$ at time $t = 0$. (Simplify as much as possible).

From above...

$$\vec{T}(0) = \frac{\langle -\sin 0,\ \sin 0 + 0\cdot\cos 0,\ e^0 \rangle}{\sqrt{\sin^2 0 + (\sin 0 + 0\cdot\cos 0)^2 + e^{2\cdot 0}}}$$

$$= \frac{\langle 0, 0, 1 \rangle}{\sqrt{0 + 0 + 1}}$$

$$= \langle 0, 0, 1 \rangle$$