

This exam has 6 problems on 6 pages. No notes, calculators, or electronic devices of any kind are allowed. **Show all your work!**

Name: _____

1. (15 points) Decide whether each of the following limits exists. If the limit exists, evaluate it.

a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+y^6}{x^2+y^2}$

b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+3xy+y^5+12}{x^6+y^6+xy^2+3}$

2. (15 points) Suppose the velocity vector of an object at time t is given by $\mathbf{v}(t) = \langle 2t, 4t^3, 3 \rangle$.

a) What is the position vector of the object at time 1 if the position vector at time 0 is given by $\mathbf{r}(0) = \langle 1, 1, 2 \rangle$?

b) What is the acceleration vector of the object at time t ?

3. (15 points) Let $f(x, y) = x^3 + \sin(xy) + 2e^y$.

a) Compute the linearization $L(x, y)$ of $f(x, y)$ at the point $(2, 0)$.

b) Find an equation for the tangent plane to the graph of f at the point $(2, 0, f(2, 0))$.

4. (20 points) Suppose the elevation of a mountain range at the point (x, y) is given by the function $f(x, y) = x^2 - xy^3 + 35y$.

a) Find the gradient vector of $f(x, y)$ at the point $(3, 2)$.

b) Suppose a hiker is standing at the point $(3, 2, f(3, 2))$. What is the rate of ascent (= slope) if the hiker starts walking in the direction of the unit vector $\mathbf{u} = \frac{1}{5}\langle 3, 4 \rangle$?

5. (20 points) Let $f(x, y) = x^5 + y^5 - 5xy$.

a) Find the critical points of f .

b) Determine whether f has a saddle point or a local maximum or minimum at each critical point found in part a.

6. (15 points) Use an iterated integral to compute the volume of the solid that lies underneath the surface $z = x + 3xy^2$ and above the square $[0,5] \times [0,1] = \{(x,y) \mid 0 \leq x \leq 5, 0 \leq y \leq 1\}$.