Exam 3 MAT 397 Section 001 Spring 2016

This exam has 6 problems on 6 pages. No notes, calculators, or electronic devices of any kind are allowed. **Show all your work!**

Name: ____

1. (15 points) Evaluate the double integral $\iint_D 2x dA$ where D is the region in $\mathbb{R}^2$ given by the equations $0 \leq x \leq 1$ and $2x^2 - x \leq y \leq 2x$.

2. (15 points) Suppose a solid object occupies the region

$$E = \{\, (x, y, z) \mid 0 \le x \le \pi/2\,,\ 0 \le y \le x,\ 0 \le y + z \le 2 \,\}.$$

Find the total mass of the object if the mass density at (x, y, z) is given by $\rho(x, y, z) = 3 + \cos y$.

3. (20 points) Use cylindrical coordinates to compute the volume $V = \iiint_E 1dV$ of the region E that lies above xy-plane, inside the cylinder $x^2 + y^2 \leq 1$, and underneath the surface $z = 2e^{x^2+y^2} + 4$.

4. (15 points) Let E be the region in $\mathbb{R}^3$ which lies above the xy-plane and between the spheres of radius 4 and 5 with centers at the origin. Use spherical coordinates to express the triple integral

$$\iiint\limits_E z\,dV$$

as an iterated integral. Do **<u>not</u>** evaluate the integral!

5. (20 points) Let $R = \{(x, y) | 0 \leq x/2 + y \leq 1\ , 0 \leq x + 3y \leq 1\}$.

a) Express x and y in terms of the variables $u = x/2 + y$ and $v = x + 3y$.

b) Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

c) Use parts a) and b) to evaluate the double integral $\iint_R \; y dA$.

6. (15 points) A lamina occupies the region

$$D = \{\, (x, y) \mid 1 \le x \le 2,\ \ 0 \le y \le 6x\}$$

in the xy-plane. Find the center of mass of the lamina if the mass density is constant equal to 1.