Solutions

Mat 397 Spring 2016 Quiz 10 Name: Caleb McWhorter Score: $\overline{10}$

1. Compute the line integral $\int_C f(x,y)\,ds$, where $f(x,y)=4x^2y$ and $\mathcal{C}$ is the curve $\mathcal{C}:\vec{r}(t)=\langle \cos 2t,\ \sin 2t\rangle,\ 0\le t\le \frac{\pi}{2}$.

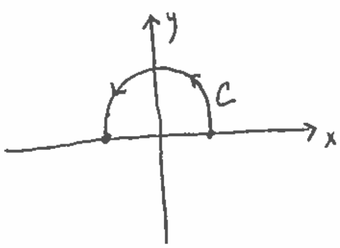
$\vec{r}(t)=\langle \cos 2t, \sin 2t\rangle$

$\vec{r}\,'(t)=\langle -2\sin 2t, 2\cos 2t\rangle$

$|\vec{r}\,'(t)|=\sqrt{4\sin^2 2t+4\cos^2 2t}$

$=\sqrt{4}$

$=2$



$\int_C 4x^2y\,ds$

$\int_0^{\frac{\pi}{2}} 4(\cos^2 2t)\sin 2t \cdot 2\,dt$

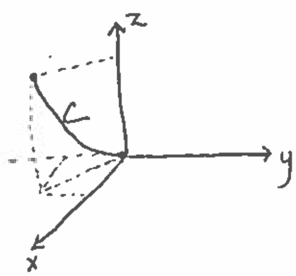
$-4\int_0^{\frac{\pi}{2}} \cos^2 2t \cdot -2\sin 2t\,dt$

$2\cdot -4 \cdot \frac{\cos^3 2t}{6}\Big|_0^{\frac{\pi}{2}}$

$-8\left[\frac{(-1)^3}{6}-\frac{1}{6}\right]$

$-8\left[\frac{-1}{6}-\frac{1}{6}\right] = -8\cdot\frac{-1}{3} = 8/3$

2. Compute the line integral $\int_C F.d\vec{r}$, where $F=\langle 3x,\ 2z,\ z-y\rangle$ and $\mathcal{C}$ is the curve $\mathcal{C}:\vec{r}(t)=\langle t,\ -t,\ t^2\rangle,\ 0\le t\le 1$.



$\vec{r}(t)=\langle t,-t,t^2\rangle$

$r'(t)=\langle 1,-1,2t\rangle$

$F(r(t))=\langle 3t, 2t^2, t^2+t\rangle$

$\int_C \vec{F}\cdot d\vec{r}$

$\int_0^1 \langle 3t, 2t^2, t^2+t\rangle\cdot\langle 1,-1,2t\rangle\,dt$

$\int_0^1 3t-2t^2+2t(t^2+t)\,dt$

$\int_0^1 3t-\cancel{2t^2}+2t^3+\cancel{2t^2}\,dt$

$\int_0^1 3t+2t^3\,dt$

$\frac{3t^2}{2}+\frac{t^4}{2}\Big|_0^1$

$\frac{3t^2+t^4}{2}\Big|_0^1 = \frac{3+1}{2} = 2$