Mat 397 Spring 2016 Quiz 2 Name: Caleb McWhorter Solutions **Score:** $\overline{10}$

1. (3 Points.) Are the following two lines parallel? Explain how you know.

$$L_1: \begin{cases} x = 3+t \\ y = 1-2t \\ z = t \end{cases} \quad -\infty < t < \infty \quad \text{and} \quad L_2: \begin{cases} x = 5-2s \\ y = 9+4s \\ z = 3-2s \end{cases} \quad -\infty < s < \infty$$

L_1 has direction ("slope") $\langle 1,-2,1\rangle$

L_2 has direction ("slope") $\langle -2,4,-2\rangle$

But notice $\langle -2,4,-2\rangle = -2\langle 1,-2,1\rangle$

So the lines are $\parallel$ so long as they are not the same line.

Since they are "$\parallel$", if they have one point in common, they are the same.

L_1 has point $(3,1,0)$ by setting $t=0$. Does L_2 contain this point? If it did.

$z = 0 = 3-2s$ so $s = 3/2$

But $x = 5-2s = 5-2(3/2) = 5-3 = 2 \neq 3$

2. (a) (4 Points.) Find the point of intersect of the lines

$$L_1: \begin{cases} x = 1+2t \\ y = 2+3t \\ z = 4+t \end{cases} \quad -\infty < t < \infty \quad \text{and} \quad L_2: \begin{cases} x = 6+s \\ y = 9+s \\ z = 3-3s \end{cases} \quad -\infty < s < \infty$$

If they have a point in common, then there is a t and s such that the following system of equations is consistent:

$$\left.\begin{aligned} x&: \quad 1+2t = 6+s \\ y&: \quad 2+3t = 9+s \\ z&: \quad 4+t = 3-3s \end{aligned}\right\} \rightarrow \begin{aligned} 2t-s&=5 \\ 3t-s&=7 \\ t+3s&=-1 \end{aligned}$$

Working with the first two equations

$$\begin{aligned} 2t-s&=5 \\ -\quad 3t-s&=7 \\ \hline -t&=-2 \\ t&=2 \end{aligned}$$

So...

$$\begin{aligned} 2t-s&=5 \\ 4-s&=5 \\ s&=-1 \end{aligned}$$

$t=2$ and $s=-1$ satisfy the first two equations but what about the third?

$$\begin{aligned} 4+t &= 3-3s \\ 4+2 &= 3-3(-1) \\ 6&=6 \ \checkmark \end{aligned}$$

$$\begin{aligned} x &= 6+s = 5 \\ y &= 9+s = 8 \\ z &= 3-3s = 6 \end{aligned}$$

The point of intersection is $(5,8,6)$

(b) (3 Points.) What is the angle between the two lines?

L_1 has direction $\langle 2,3,1\rangle$

L_2 has direction $\langle 1,1,-3\rangle$

$$\vec{u}\cdot\vec{v} = |\vec{u}|\,|\vec{v}|\cos\theta$$

$$\langle 2,3,1\rangle\cdot\langle 1,1,-3\rangle = |\langle 2,3,1\rangle|\,|\langle 1,1,-3\rangle|\cos\theta$$

$$2(1)+3(1)+1(-3) = \sqrt{2^2+3^2+1^2}\sqrt{1^2+1^2+3^2}\cos\theta$$

$$2 = \sqrt{14}\,\sqrt{11}\cos\theta$$

$$\cos\theta = 2/\sqrt{14}\sqrt{11}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{14}\sqrt{11}}\right) \approx 80.73^\circ$$