Solutions

Mat 397 Spring 2016 Quiz 4 Name: Caleb McWhorter Score: $\overline{10}$

1. Let $\mathbf{r}(t) = \sin 2t\,\mathbf{i} - \cos 2t\,\mathbf{j} + t\,\mathbf{k}$.

(a) Find the unit tangent vector $\mathbf{T}(t)$.

$\vec{r}(t) = \langle \sin 2t, -\cos 2t, t \rangle$

$\vec{r}\,'(t) = \langle 2\cos 2t, 2\sin 2t, 1 \rangle$

$|\vec{r}\,'(t)| = \sqrt{(2\cos 2t)^2 + (2\sin 2t)^2 + 1^2}$

$= \sqrt{4\cos^2 2t + 4\sin^2 2t + 1}$

$= \sqrt{4+1}$

$= \sqrt{5}$

$\vec{T}(t) = \dfrac{r'(t)}{|r'(t)|}$

$= \dfrac{\langle 2\cos 2t, 2\sin 2t, 1 \rangle}{\sqrt{5}}$

$= \left\langle \frac{2}{\sqrt{5}}\cos 2t, \frac{2}{\sqrt{5}}\sin 2t, 1/\sqrt{5} \right\rangle$

(b) Find the principal normal vector $\mathbf{N}(t)$.

$\vec{T}\,'(t) = \left\langle \frac{-4}{\sqrt{5}}\sin 2t, \frac{4}{\sqrt{5}}\cos 2t, 0 \right\rangle$

$|T'(t)| = \sqrt{\left(\frac{-4}{\sqrt{5}}\sin 2t\right)^2 + \left(\frac{4}{\sqrt{5}}\cos 2t\right)^2 + 0^2}$

$= \sqrt{\frac{16}{5}\sin^2 2t + \frac{16}{5}\cos^2 2t}$

$= \sqrt{16/5}$

$= 4/\sqrt{5}$

$\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{|T'(t)|}$

$\vec{N}(t) = \dfrac{\left\langle \frac{-4}{\sqrt{5}}\sin 2t, \frac{4}{\sqrt{5}}\cos 2t, 0 \right\rangle}{4/\sqrt{5}}$

$= \left\langle \frac{\sqrt{5}}{4}\frac{-4}{\sqrt{5}}\sin 2t, \frac{\sqrt{5}}{4}\frac{4}{\sqrt{5}}\cos 2t, 0 \right\rangle$

$= \langle -\sin 2t, \cos 2t, 0 \rangle$

(c) Find the length of the curve C: $\mathbf{r}(t) = \sin 2t\,\mathbf{i} - \cos 2t\,\mathbf{j} + t\,\mathbf{k}$; $0 \le t \le 3$.

$\vec{r}(t) = \langle \sin 2t, -\cos 2t, t \rangle$

$\vec{r}\,'(t) = \langle 2\cos 2t, 2\sin 2t, 1 \rangle$

$|r'(t)| = \sqrt{(2\cos 2t)^2 + (2\sin 2t)^2 + 1^2}$

= via work in (a)

$= \sqrt{5}$

$L = \int_0^3 |r'(t)|\,dt$

$= \int_0^3 \sqrt{5}\,dt$

$= \sqrt{5}\int_0^3 dt$

$= 3\sqrt{5}$