

Solutions

Mat 397 Spring 2016 Quiz 5

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Score:
10

1. For each of the following limits either compute its value, or show that it does not exist

(a) $\lim_{(x,y) \rightarrow (2,3)} \frac{x^2 + 2y^2}{x^2 - y + 1}$

This is continuous in its domain - which includes $(2,3)$. So

$$\lim_{(x,y) \rightarrow (2,3)} \frac{x^2 + 2y^2}{x^2 - y + 1} = \frac{2^2 + 2(3)^2}{2^2 - 3 + 1} = \frac{4 + 18}{2} = 11$$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 3y^2}{x^2 + y^2}$

x-axis ($y=0$): $\lim_{(x,0) \rightarrow (0,0)} \frac{x^2 + 0}{x^2 + 0} = 1$

y-axis ($x=0$): $\lim_{(0,y) \rightarrow (0,0)} \frac{0 + 3y^2}{0 + y^2} = 3$

Therefore, the limit does not exist.

2. If $f(x,y) = xe^{xy} + \sin x$ compute the following:

(a) $\frac{\partial f}{\partial x} = xye^{xy} + e^{xy} + \cos x = e^{xy}(1 + xy) + \cos x$

(b) $\frac{\partial^2 f}{\partial y \partial x} = xe^{xy}(1 + xy) + xe^{xy} = xe^{xy}(2 + xy)$

OR

$$\begin{aligned} \frac{\partial}{\partial y} (xye^{xy} + e^{xy} + \cos x) &= xe^{xy} + x^2ye^{xy} + xe^{xy} \\ &= 2xe^{xy} + x^2ye^{xy} \end{aligned}$$