Solutions

Mat 397 Spring 2016 Quiz 6 Name: Caleb McWhorter **Score:** $\frac{\quad}{10}$

1. (a) Compute the directional derivative $(D_u f)_P$ when $f(x,y,z) = y + 2ze^{xy}$, P is the point $(2,0,3)$ and $\mathbf{u} = \frac{1}{\sqrt{6}}\langle 1, -2, 1\rangle$.

$f(x,y,z) = y + 2ze^{xy}$

$\nabla f(x,y,z) = \langle 2yze^{xy},\ 1 + 2xze^{xy},\ 2e^{xy}\rangle$

$\nabla f(2,0,3) = \langle 0, 13, 2\rangle$

$|\vec{u}| = \frac{1}{\sqrt{6}}\sqrt{1^2+2^2+1^2} = \frac{\sqrt{6}}{\sqrt{6}} = 1$

$$D_u f(2,0,3) = \nabla f(2,0,3) \cdot \vec{u}$$
$$= \langle 0,13,2\rangle \cdot \frac{\langle 1,-2,1\rangle}{\sqrt{6}}$$
$$= \frac{0(1) + 13(-2) + 2(1)}{\sqrt{6}}$$
$$= \boxed{-24/\sqrt{6}}$$

(b) Find the equation of the tangent plane to the level surface $f(x,y,z) = y + 2ze^{xy} = 6$ at the point $P(2,0,3)$ from part (a).

$f(x,y,z) = y + 2ze^{xy} - 6$

$\nabla f = \langle 2yze^{xy},\ 1 + 2xze^{xy},\ 2e^{xy}\rangle$

$\nabla f(2,0,3) = \langle 0, 13, 2\rangle$

So the tangent plane is...

$$\langle 0,13,2\rangle \cdot \langle x-2, y-0, z-3\rangle = 0$$
$$0(x-2) + 13(y-0) + 2(z-3) = 0$$
$$13y + 2z - 6 = 0$$
$$\boxed{13y + 2z = 6}$$