Solutions

Find all critical points for the function $f(x,y) = 2x^2 + 2xy + y^2 + 2x - 3$ and use the second derivative test to classify them.

$$\begin{cases} f_x = 4x + 2y + 2 = 0 \\ f_y = 2x + 2y = 0 \end{cases}$$

Subtract bottom from top and you get....

$$2x + 2 = 0$$
$$2(x+1) = 0$$

So $x = -1$.

$$2x + 2y = 0$$
$$2(-1) + 2y = 0$$
$$-2 + 2y = 0$$
$$2(y-1) = 0$$

So $y = 1$

To classify, we look at....

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$$f_{xx} = 4$$
$$f_{yy} = 2$$
$$f_{xy} = f_{yx} = 2$$

So at $(-1, 1)$, we have

$$\begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} = 8 - 4 > 0$$

$f_{xx} = 4 > 0$ so $(-1,1)$ is a minimum.