Solutions

Mat 397 Spring 2016 **Bonus Quiz!** Name: Caleb McWhorter Score: $\frac{\quad}{10}$

Find the maximum and minimum values of the function $f(x,y)=x^2y$ on the curve $g(x,y) \overset{def}{=} x^2+y^2=4$.
[**Hint:** After moving all terms to the left, factor the first of the three equations you get. This will lead to two possibilities. Find the points associated with each possibility separately. One of these possibilities is easy to solve. The other needs you to use the second equation to find a relationship between x^2 and y^2. Do your algebra carefully and use back if you need extra space.]

$\nabla f = \langle 2xy, x^2 \rangle$

$\nabla g = \langle 2x, 3y \rangle$

$$\begin{cases} 2xy = 2x\lambda \\ x^2 = 2y\lambda \\ x^2+y^2=4 \end{cases}$$

$2xy - 2x\lambda = 0$

$2x(y-\lambda)=0$

$2x=0$ or $y-\lambda=0$

$x=0$ $\quad y=\lambda$

If $x=0$: $x^2+y^2=4$

$y^2=4$

$y=\pm 2$

$(0,2)$, $(0,-2)$

If $y=\lambda$: $x^2=2y\lambda$

$x^2=2y\cdot y$

$x^2=2y^2$

Then... $x^2+y^2=4$

$2y^2+y^2=4$

$3y^2=4$

$y^2=4/3$

$y=\pm 2/\sqrt{3}$

Then...

$x^2+y^2=4$

$x^2+\left(\pm\frac{2}{\sqrt{3}}\right)^2=4$

$x^2+\frac{4}{3}=4$

$x^2=4-\frac{4}{3}$

$x^2=\frac{12-4}{3}$

$x^2=\frac{8}{3}$

$x=\pm\sqrt{\frac{8}{3}}$

So we have points:

$\left(\pm\sqrt{8/3}, \pm 2/\sqrt{3}\right)$

$(0, \pm 2)$

$f(0,\pm 2)=0$

$f\left(\pm\sqrt{8/3}, 2/\sqrt{3}\right)=\frac{16}{3\sqrt{3}}$ max

$f\left(\pm\sqrt{8/3}, -2/\sqrt{3}\right)=\frac{-16}{3\sqrt{3}}$ min.