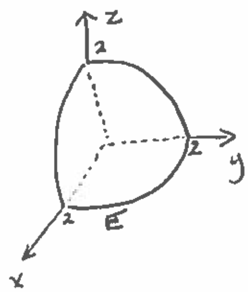
Solutions

Mat 397 Spring 2016 Quiz 5 Name: Caleb McWhorter **Score:** $\frac{}{10}$

1. Write the integral $\iiint_E xz\,dV$, where E is the finite region in the first octant bounded by the coordinate planes and $x^2+y^2+z^2=4$, as an iterated integral using spherical coordinates. (You need not evaluate the integral.)



$$\iiint_E xz\,dV = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^2 \rho\sin\phi\cos\theta\cdot\rho\cos\phi\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^2 \rho^4\sin^2\phi\cos\phi\cos\theta\,d\rho\,d\phi\,d\theta$$

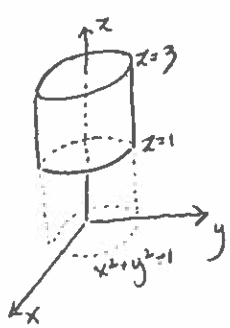
$$= \int_0^{\frac{\pi}{2}}\cos\theta\,d\theta\cdot\int_0^{\frac{\pi}{2}}\sin^2\phi\cos\phi\,d\phi\cdot\int_0^2\rho^4\,d\rho$$

$$= \sin\theta\Big|_0^{\frac{\pi}{2}}\cdot\frac{\sin^3\phi}{3}\Big|_0^{\frac{\pi}{2}}\cdot\frac{\rho^5}{5}\Big|_0^2$$

$$= 1\cdot\frac{1}{3}\cdot\frac{32}{5}$$

$$= 32/15$$

2. Use cylindrical coordinates to compute the moment of inertia about the z-axis of the region inside cylinder $x^2+y^2=1$ and between the planes $z=1$ and $z=3$ if the density is given by $\rho(x,y,z)=2(x^2+y^2)$.



$$I_z = \iiint_E (x^2+y^2)\rho(x,y,z)\,dV$$

$$= \int_0^{2\pi}\int_0^1\int_1^3 r^2\cdot 2r^2\cdot r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1\int_1^3 2r^5\,dz\,dr\,d\theta$$

$$= 2\cdot\int_0^1 r^5\,dr\cdot\int_1^3 dz\cdot\int_0^{2\pi} d\theta$$

$$= 2\cdot\frac{r^6}{6}\Big|_0^1\cdot(3-1)\cdot 2\pi$$

$$= \cancel{2}^{1}\cdot\frac{1}{\cancel{6}_{3}}\cdot 2\cdot 2\pi$$

$$= \frac{4\pi}{3}$$