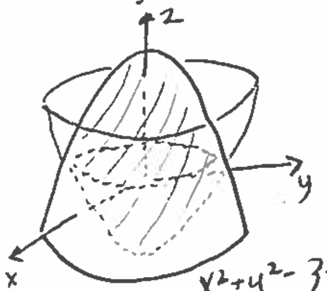
Math 397 Spring 2016 Solutions Quiz 20 Name Caleb McWhorter

Show all work. Incomplete answers may receive little or no credit. You do not need to simplify your answers. All numerical answers should be exact, with no decimal approximations.

1. Suppose E is the solid bounded below by the paraboloid $z = x^2 + y^2 - 3$ above the paraboloid $z = 9 - 2(x^2 + y^2)$.

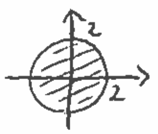
(a) Write down a triple integral in **cylindrical coordinates** for M_{xy}, the moment about the xy-plane, if the density function is 1. Do **NOT** integrate. (Hint: Find the intersection of the two paraboloids.)



$x^2+y^2-3 = 9-2(x^2+y^2)$

$3x^2+3y^2 = 12$

$x^2+y^2 = 4$



$$M_{xy} = \iiint_E z\,\rho(x,y,z)\,dV$$

$$= \int_0^{2\pi}\int_0^2\int_{r^2-3}^{9-2r^2} z \cdot r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2\int_{r^2-3}^{9-2r^2} zr\,dz\,dr\,d\theta = 56\pi$$

(b) Assume the integral from part (a) yields $M_{xy} = 56\pi$. Use this fact, symmetry, and the fact that the mass of the solid is $m = 24\pi$ to write down the centroid for the solid E.

The region E is symmetric about the z-axis. This gives $\bar{x}=0$ and $\bar{y}=0$. We know $\bar{z} = \frac{M_{xy}}{m}$ Total mass so $\bar{z} = \frac{56\pi}{24\pi} = 7/3$. So the centroid is $(0,0,7/3)$

2. Draw a clear sketch of the following surfaces in cylindrical coordinates. Identify what the surface is.

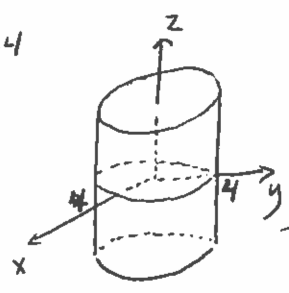
(a) $r = 4$

$\Updownarrow$

$\sqrt{x^2+y^2}=4$ for all z.

$\Updownarrow$

$x^2+y^2=16$ for all z

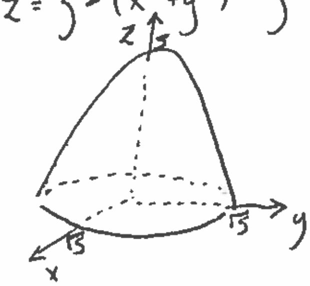


This is a cylinder

(b) $z = 5 - r^2$

$\Updownarrow$

$z = 5-(x^2+y^2) = 5-x^2-y^2$



This is a paraboloid

3. Rewrite the equation $z = \sqrt{x^2 + y^2}$ in cylindrical coordinates. Sketch & identify the surface.

$z = \sqrt{x^2+y^2}$

$z = \sqrt{r^2}$

$z = |r|$

But $r \geq 0$ by definition. So

$z = r$

This is a cone.