Solutions

Math 397 Spring 2016
Section 4

Quiz 22

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Show all work. Incomplete answers may receive little or no credit. You need not simplify your answers. All numerical answers should be exact, with no decimal approximations.

1. Consider the integral $I = \iint_R (x+y)\,dA$ where R is the region in the xy-plane bounded by the lines $y-3x=2$, $y-3x=8$, $x+y=3$, and $x+y=6$. Answer the following questions using the change of variables $u=y-3x$ and $v=x+y$.

(a) Solve for x and y in terms of u and v.

$$\begin{cases} u = y-3x \\ v = y+x \end{cases}$$

$$\begin{cases} u = y-3x \\ 3v = 3y+3x \end{cases}$$

Add

$$u+3v = 4y$$

$$y = \frac{u+3v}{4}$$

$$v = y+x$$

$$4v = 4y + 4x$$

$$4v = 4\cdot\frac{u+3v}{4} + 4x$$

$$4v = u+3v+4x$$

$$-4x = u-v$$

$$x = \frac{u-v}{-4}$$

$$x = \frac{v-u}{4}$$

$x =$ $\frac{v-u}{4}$

$y =$ $\frac{u+3v}{4}$

(b) Find $J(u,v)$

$$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} -\frac{1}{4} & 1/4 \\ 1/4 & 3/4 \end{vmatrix} = -\frac{3}{16} - \frac{1}{16} = \frac{-4}{16} = -1/4$$

$J(u,v) =$ $-1/4$

(c) Rewrite **but do not integrate** the integral I as a double iterated integral, complete with limits of integration, in terms of u and v.

$$\iint_R x+y\,dA = \int_3^6\int_2^8 v\left|\tfrac{-1}{4}\right|du\,dv = \frac{1}{4}\int_3^6\int_2^8 v\,du\,dv$$

2. Suppose a region of integration in the first quadrant is bounded by the curves $y=\frac{2}{x}$, $y=\frac{7}{x}$, $x=y^2+1$, and $x=y^2+3$. Give a suitable change of variables and the corresponding limits of integration for this region.

$\to xy=2$, $\to xy=7$

$x-y^2=1$, $x-y^2=3$

$u =$ xy limits of integration for u: $2 \,\&\, 7$

$v =$ $x-y^2$ limits of integration for v: $1 \,\&\, 3$