Solutions

Math 397 Spring 2016 **Quiz 3** Name Caleb McWhorter

Section 4

Show all work. Incomplete answers may receive little or no credit. You need not simplify your answers. All numerical answers should be exact, with no decimal approximations.

1. Consider the vectors $\mathbf{u} = \langle 4, 1, 3\rangle$ and $\mathbf{v} = \langle -2, 3, 4\rangle$

(a) Find $\mathbf{u} \cdot \mathbf{v}$

$$\vec{u}\cdot\vec{v} = 4(-2) + 1(3) + 3(4)$$
$$= -8 + 3 + 12$$
$$= 7$$

(b) Find θ, the angle between the two vectors.

We know $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$$|\vec{u}| = \sqrt{4^2+1^2+3^2} = \sqrt{16+1+9} = \sqrt{26}$$
$$|\vec{v}| = \sqrt{(-2)^2+3^2+4^2} = \sqrt{4+9+16} = \sqrt{29}$$

So, $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$$7 = \sqrt{26}\sqrt{29}\cos\theta$$
$$\cos\theta = \frac{7}{\sqrt{26}\sqrt{29}}$$
$$\theta = \cos^{-1}\left(\frac{7}{\sqrt{26}\sqrt{29}}\right)$$

2. Are the vectors $\mathbf{a} = \langle 1, 2, -3\rangle$ and $\mathbf{b} = \langle -4, 5, 2\rangle$ orthogonal? Show why or why not.

Two vectors are orthogonal if and only if $\vec{u}\cdot\vec{v} = 0$

$\vec{a}\cdot\vec{b} = 1(-4) + 2(5) + (-3)2 = -4 + 10 + -6 = 0$ so these are orthogonal.

3. Write down a nonzero vector that is orthogonal to $\mathbf{v} = \langle 2, -1, 5\rangle$. Hint: There are infinitely many correct answers.

As above, two vectors are orthogonal if and only if their dot product is zero. Let $\vec{a} = \langle x, y, z\rangle$

$$\vec{v}\cdot\vec{a} = 2x - y + 5z = 0$$

Any (x,y,z) making this work will do.

$\langle 3,6,0\rangle$, $\langle -2,1,1\rangle$, $\langle 0,5,1\rangle$, $\langle -5,0,2\rangle$

$\langle 5,0,-2\rangle$, $\langle 1,2,0\rangle$, etc.