Math 397 Spring 2016 **Quiz 9** Name Caleb McWhorter

Section 4

Show all work. Incomplete answers may receive little or no credit.

1. Find r(t) if $\mathbf{r}'(t) = 2\mathbf{i} + 6t\mathbf{j} - 4\sin t\,\mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ at time $t = 0$.

$$\vec{r}(t) = \int \vec{r}'(t)\,dt$$

$$= \int \langle 2, 6t, -4\sin t \rangle\,dt$$

$$= \langle 2t, 3t^2, 4\cos t \rangle + \langle C_1, C_2, C_3 \rangle$$

So $\vec{r}(0) = \langle 0, 0, 4 \rangle + \langle C_1, C_2, C_3 \rangle = \langle 1, 2, 1 \rangle$

$C_1 = 1$

$C_2 = 2$

$4 + C_3 = 1 \to C_3 = -3$

$$\vec{r}(t) = \langle 2t + 1, 3t^2 + 2, 4\cos t - 3 \rangle$$

2. Suppose a curve in 3-space is given by $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ $= \langle 1, t^2, t^3 \rangle$

(a) Find $\mathbf{T}(t)$ for the curve above.

$$\vec{r}'(t) = \langle 0, 2t, 3t^2 \rangle$$

$$|r'(t)| = \sqrt{4t^2 + 9t^4} = \sqrt{t^2(4 + 9t^2)} = t\sqrt{4 + 9t^2}$$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \boxed{\frac{\langle 0, 2t, 3t^2 \rangle}{\sqrt{4t^2 + 9t^4}}} = \frac{\langle 0, 2t, 3t^2 \rangle}{t\sqrt{4 + 9t^2}} = \frac{\langle 0, 2, 3t \rangle}{\sqrt{4 + 9t^2}}$$

(b) State the formula you would use to find $\mathbf{N}(t)$. **Do not actually find $\mathbf{N}(t)$!**

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|}$$

(c) State the formula you would use to find $\mathbf{B}(t)$. **Do not actually find $\mathbf{B}(t)$!**

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

(d) Find the length of the curve for $0 \le t \le 2$. Note: You have already done a lot of the work.

$$AL = \int_0^2 |r'(t)|\,dt$$

From (a), $|r'(t)| = t\sqrt{4 + 9t^2}$

$$\int_0^2 t\sqrt{4 + 9t^2}\,dt = \int_4^{40} t\sqrt{u}\,\frac{du}{18t} = \frac{1}{18}\int_4^{40} u^{1/2}\,du = \frac{1}{18}\cdot\frac{2}{3}u^{3/2}\Big|_4^{40}$$

$$= \frac{1}{27}u^{3/2}\Big|_4^{40}$$

$u = 4 + 9t^2$

$du = 18t\,dt$

$dt = \frac{du}{18t}$

$u(0) = 4$

$u(2) = 40$

$$= \frac{1}{27}\left(40^{3/2} - 4^{3/2}\right)$$

$$= 1/27\left(\sqrt{64000} - 8\right)$$

$$= \frac{8}{27}\left(10\sqrt{10} - 1\right)$$