Solutions

Math 397 Spring 2016 **Quiz 13** Name Caleb McWhorter

Section 4 Circle ~~lecture~~ time: 9:30 11:00

Show all work. Incomplete answers may receive little or no credit. You need not simplify your answers.

1. (a) Find the linearization of $f(x,y) = \cos \pi x + xy^2$ at the point (2,1).

$f(x,y) = \cos \pi x + xy^2$

$f(2,1) = \cos 2\pi + 2 = 3$

$\nabla f = \langle -\pi \sin \pi x + y^2, 2xy \rangle$

$\nabla f(2,1) = \langle 1, 4 \rangle$

$L(x,y) = z_0 + f_x (x - x_0) + f_y (y - y_0)$

$L(x,y) = 3 + (x-2) + 4(y-1)$

(b) **Use your answer** to part (a) to **estimate** $f(1.9,1.2)$. **You do not need a calculator for this.** (Note: My calculator gives $f(1.9,1.2) = 3.731$)

$$f(1.9, 1.2) \approx L(1.9, 1.2) = 3 + (1.9-2) + 4(1.2-1)$$
$$= 3 - 0.1 + 4(.2)$$
$$= 3 - 0.1 + .8$$
$$= 3.7$$

2. Let $z = x^2 y + \cos y - 5x$ while $x = t^3 + s^3$ and $y = s^2 e^t$.

(a) Write down the chain rule **formula** for $\frac{\partial z}{\partial t}$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = z_x x_t + z_y y_t$$

(b) Use the chain rule to find $\frac{\partial z}{\partial t}$. (Give your answer in terms of s and t, but **do not simplify** at all.)

$z_x = 2xy - 5$

$z_y = x^2 - \sin y$

$x_t = 3t^2$

$y_t = s^2 e^t$

$$\frac{\partial z}{\partial t} = (2xy - 5)\, 3t^2 + (x^2 - \sin y)\, s^2 e^t$$
$$= \left(2(t^3+s^3)(s^2e^t) - 5\right) 3t^2 + \left((t^3+s^3)^2 - \sin(s^2e^t)\right) s^2 e^t$$