Solutions

Math 397 Spring 2016 **Quiz 15** Name Caleb McWhorter

Section 4

Show all work. Incomplete answers may receive little or no credit. You need not simplify your answers. All numerical answers should be exact, with no decimal approximations.

1. Find all critical points for $f(x,y) = -3xy + x^3 - y^3 + 3$ and then use the Second Derivative Test to classify them as local maxima, local minima, or saddle points.

$$\begin{cases} f_x = -3y + 3x^2 = 0 \\ f_y = -3x - 3y^2 = 0 \end{cases}$$

$$\begin{cases} 3x^2 = 3y \\ -3y^2 = 3x \end{cases}$$

$$\begin{cases} x^2 = y \\ -y^2 = x \end{cases}$$

$x^2 = y$

$(-y^2)^2 = y$

$y^4 = y$

$y^4 - y = 0$

$y(y^3 - 1) = 0$

$y = 0$ or $y^3 - 1 = 0$

$y = 1$

If $y = 0 \longrightarrow x^2 = y$, $x^2 = 0$, $x = 0$

$(0,0)$

If $y = 1 \longrightarrow x^2 = y$, $x^2 = 1$, $x = \pm 1$

$(1,1)$ & $(-1,1)$

$f_{xx} = 6x$

$f_{yy} = -6y$

$f_{xy} = f_{yx} = -3$

$$d = \begin{vmatrix} 6x & -3 \\ -3 & -6y \end{vmatrix} = -36xy - 9$$

At $\underline{(0,0)}$:

$d = 0 - 9 = -9 < 0$

So $(0,0)$ is a saddle

At $\underline{(1,1)}$:

$d = -36 - 9 = -45 < 0$

So $(1,1)$ is a saddle

At $\underline{(-1,1)}$:

$d = -36(-1)1 - 9 = 36 - 9 = 27 > 0$

$f_{xx} = 6(-1) = -6 < 0$

So $(-1,1)$ is a maximum