

Solutions (1)

Quiz 1

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1. Determine whether the points lie on a straight line.

$$A(1, 2, 1), \quad B(7, 3, -2), \quad C(2, 3, 3)$$

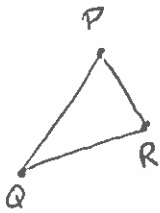
If these lie along the same line, then the 'slope' vectors we can create must be parallel.

$$\begin{aligned} \vec{AB} &= \langle 7, 3, -2 \rangle - \langle 1, 2, 1 \rangle = \langle 6, 1, -3 \rangle \\ \vec{AC} &= \langle 2, 3, 3 \rangle - \langle 1, 2, 1 \rangle = \langle 1, 1, 2 \rangle \end{aligned} \quad \left. \vphantom{\begin{aligned} \vec{AB} \\ \vec{AC} \end{aligned}} \right\} \begin{array}{l} \text{We need to check if these} \\ \text{are parallel.} \end{array}$$

$$\begin{aligned} \langle 6, 1, -3 \rangle \\ \times 6 \uparrow \quad \times 1 \uparrow \quad 1 \times \frac{-3}{2} \rightarrow \\ \langle 1, 1, 2 \rangle \end{aligned}$$

Remember $\vec{a} \parallel \vec{b}$ if there is a number $c \neq 0$ so $\vec{a} = c\vec{b}$. To get from 1 to 6, we multiply by 6. If these are \parallel , it must be 6 for each but it is not. So they are not \parallel , so the points are not colinear.

2. Determine whether the triangle with vertices $P = (2, -1, 0)$, $Q(4, 0, -1)$ and $R(3, 1, 4)$ is right-angled.



If a $\triangle PQR$ is right, then one of the \angle 's is 90° . Then two of the vectors making up the triangle are \perp . We form these vectors and test orthogonality.

$$PQ = \langle 2, -1, 0 \rangle - \langle 4, 0, -1 \rangle = \langle -2, -1, 1 \rangle$$

$$RQ = \langle 3, 1, 4 \rangle - \langle 4, 0, -1 \rangle = \langle -1, 1, 5 \rangle$$

$$PR = \langle 2, -1, 0 \rangle - \langle 3, 1, 4 \rangle = \langle -1, -2, -4 \rangle$$

$$\begin{aligned} PQ \cdot RQ &= -2(-1) + -1(1) + 1(5) = 6 \neq 0 \\ PR \cdot RQ &= -1(-1) + -2(1) + -4(5) = -21 \neq 0 \\ PQ \cdot PR &= -2(-1) + -1(-2) + 1(-4) = 0 \end{aligned}$$

So $PQ \cdot PR$ is 0 so PQ, PR are orthogonal. So $\angle QPR$ is a right angle. So $\triangle PQR$ is a right triangle.

3. Describe in words the region in space represented by the inequality.

$$0 \leq z \leq 5$$

This is the set of all points (x, y, z) such that $0 \leq z \leq 5$. So this "looks like" an infinite box with height 5 sitting on the xy -plane.

