Solutions

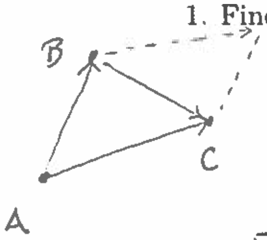
MAT 397 CAL III Section M005 Spring 2016

Quiz 2

Your Name (please PRINT): Caleb McWhorter

Student ID Number: ______________________

1. Find the area of the triangle with vertices $A(-2,1,3)$, $B(2,2,-1)$ and $C(3,7,3)$.



$\vec{AB} = \langle 2,2,-1\rangle - \langle -2,1,3\rangle = \langle 2-{-2},\ 2-1,\ -1-3\rangle = \langle 4,1,-4\rangle$

$\vec{AC} = \langle 3,7,3\rangle - \langle -2,1,3\rangle = \langle 3-{-2},\ 7-1,\ 3-3\rangle = \langle 5,6,0\rangle$

$$\vec{AB}\times\vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -4 \\ 5 & 6 & 0 \end{vmatrix} = i\,(0 - {-24}) - j\,(0 - {-20}) + \hat{k}\,(24 - 5)$$
$$= 24\hat{i} - 20\hat{j} + 19\hat{k} = \langle 24,-20,19\rangle$$

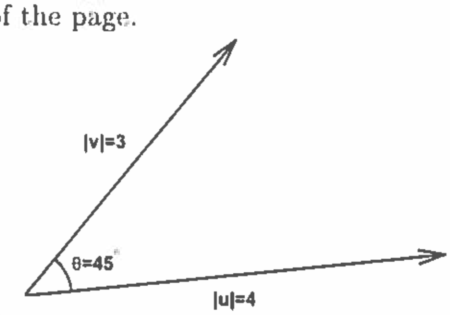
Recall $|\vec{AB}\times\vec{AC}|$ is the area of the ▱ 'spanned' by $\vec{AB},\vec{AC}$ via the ▱-law. The area of the △ is half this.

$$|\vec{AB}\times\vec{AC}| = \sqrt{24^2+20^2+19^2} = \sqrt{576+400+361} = \sqrt{1337}$$

$$\text{So area of } \triangle ABC = \frac{\sqrt{1337}}{2}$$

2. Vectors **u** and **v** are given as in the figure below. Find $|\mathbf{u}\times\mathbf{v}|$ and determine whether $\mathbf{u}\times\mathbf{v}$ is directed into the page or out of the page.

$$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta$$
$$= 4\cdot 3\cdot \sin 45^\circ$$
$$= 4\cdot 3\cdot \tfrac{1}{\sqrt{2}}$$
$$= \frac{12}{\sqrt{2}}$$



$\vec{u}\times\vec{v}$ ⊙

Out of page

3. Is the line through $(-2, 4, 0)$ and $(1, 1, 1)$ parallel to the line through $(2, 3, 4)$ and $(3, -1, -8)$?

The line through $(-2,4,0)$ & $(1,1,1)$ has 'slope'

$m_1 = \langle -2,4,0\rangle - \langle 1,1,1\rangle = \langle -3,3,-1\rangle$

The line through $(2,3,4)$ & $(3,-1,-8)$ has 'slope'

$m_2 = \langle 2,3,4\rangle - \langle 3,-1,-8\rangle = \langle -1,4,12\rangle$

We could calculate $\vec{m}_1 \times \vec{m}_2$ and see if we get $\vec{0}$, if so they are parallel (so long as the lines are distinct). But easier... vectors $\vec{u}, \vec{v}$ are parallel if and only if there is a $c \neq 0$ so that $\vec{u} = c\vec{v}$ ...

$$\begin{array}{c} \langle -1, \quad 4, \quad 12\rangle \\ \times 3 \downarrow \quad \times\tfrac{3}{4} \downarrow \quad \downarrow \times -\tfrac{1}{12} \\ \langle -3, \quad 3, \quad -1\rangle \end{array}$$

Notice these are not the same so these lines are not parallel.