MAT 397 CAL III Section M005 Spring 2016

Quiz 3 Solutions

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1. The position function of a moving particle is $\mathbf{r}(t) = e^t\hat{\mathbf{i}} + t\hat{\mathbf{j}} - \sqrt{t}\mathbf{k}$. Find its velocity and acceleration at $t = 1$.

$\vec{r}(t) = \langle e^t, t, -\sqrt{t} \rangle$

$\vec{r}\,'(t) = \langle e^t, 1, \frac{-1}{2\sqrt{t}} \rangle$

$\vec{r}\,''(t) = \langle e^t, 0, \frac{1}{4\sqrt{t^3}} \rangle$

So

$\vec{r}\,'(1) = \langle e^1, 1, \frac{-1}{2\sqrt{1}} \rangle = \langle e, 1, -1/2 \rangle$

$\vec{r}\,''(1) = \langle e^1, 0, \frac{1}{4\sqrt{1^3}} \rangle = \langle e, 0, 1/4 \rangle$

2. Determine whether the limit exists. If it does, find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

Along x-axis, $y=0$: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ x\text{-axis}}} \frac{2xy}{x^2+y^2} = \lim_{\substack{(x,y)\to(0,0)\\ x\text{-axis}}} \frac{2x\cdot 0}{x^2+0^2} = 0$

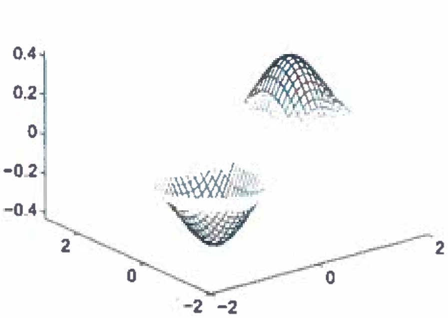
Along y-axis, $x=0$: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y\text{-axis}}} \frac{2xy}{x^2+y^2} = \lim_{\substack{(x,y)\to(0,0)\\ y\text{-axis}}} \frac{2\cdot 0\cdot y}{0^2+y^2} = 0$

Along $y=x$: $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{2xy}{x^2+y^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{2x^2}{2x^2} = \lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{2}{2} = 2/2 = 1$

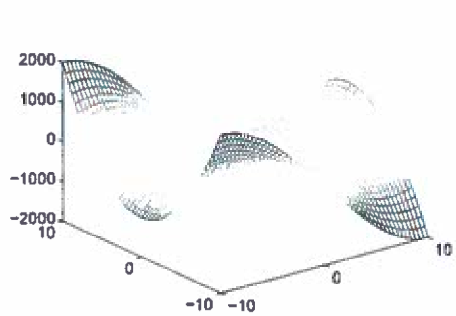
Therefore, the limit does not exist.

3. Match each of the given functions with its graph and level curves. Give reasons for your choices.

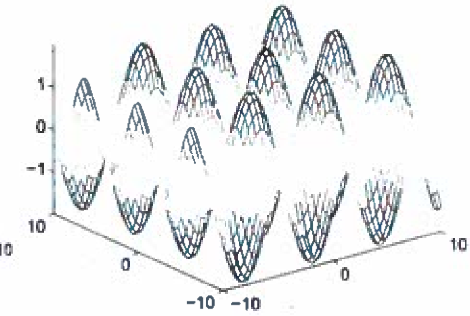
$$\underbrace{f(x,y) = xe^{-x^2-y^2}}_{i}, \qquad \underbrace{g(x,y) = x^3 - 3xy^2}_{ii}, \qquad \underbrace{h(x,y) = \sin x + \cos y}_{iii}$$



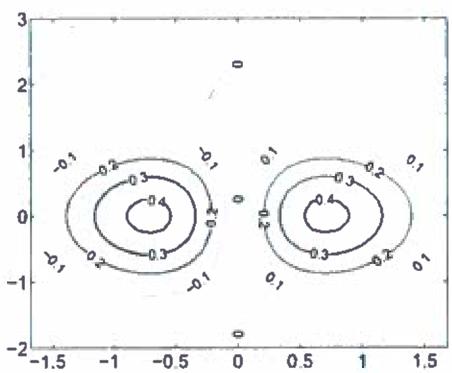
(a)



(b)



(c)



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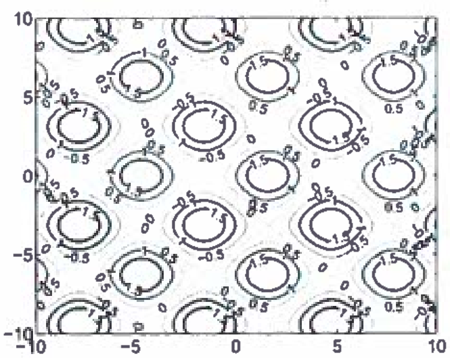
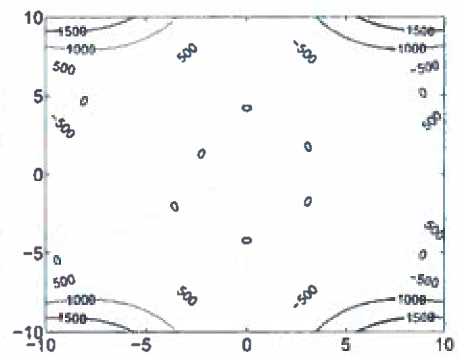


(e)



(f)

i) Graph = a
Level curves = d

We should see circular nearly symmetric level curves in z-slices

ii) Graph: b
Level Curves: f

This function is 'hyperbolic' so we should see a twisted sheet graphical appearance with hyperbolic level curves.

iii) Graph: c
Level Curves: e

This function is periodic in x or y, so we should see repeating patterns of rising/falling level curves.