MAT 397 CAL III Section M005 Spring 2016

Solutions

Quiz 5

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1. Find the critical points to the given function and use the second derivative test to determine whether they are local minimum, local maximum or saddle points.

$$f(x,y) = x^3 - 6xy + 8y^3$$

$f_x = 3x^2 - 6y$

$f_y = -6x + 24y^2$

We want $f_x = 0 = f_y$

$\begin{cases} 3x^2 - 6y = 0 \\ -6x + 24y^2 = 0 \end{cases}$

$\begin{cases} 3x^2 = 6y \\ 24y^2 = 6x \end{cases}$

$\begin{cases} x^2 = 2y \\ 4y^2 = x \end{cases}$

Now...

$4y^2 = x$

$16y^4 = x^2$

But $x^2 = 2y$ so

$16y^4 = x^2$

$16y^4 = 2y$

$16y^4 - 2y = 0$

$2y(8y^3 - 1) = 0$

Now...

$2y(8y^3 - 1) = 0$

$2y = 0$ or $8y^3 - 1 = 0$

$y = 0$

$8y^3 = 1$

$y^3 = 1/8$

$y = \sqrt[3]{1/8}$

$y = 1/2$

If $y = 0$, then

$x^2 = 2y = 0$

So $x = 0$. Gives point $(0,0)$.

If $y = 1/2$, then

$x^2 = 2y$

$x^2 = 1$

$x = \pm 1$

Gives points $(1, 1/2)$ & $(-1, 1/2)$

We look at $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

$f_{xx} = 6x$

$f_{yy} = 48y$

$f_{xy} = f_{yx} = -6$

At $(0,0)$:

$\begin{vmatrix} 0 & -6 \\ -6 & 0 \end{vmatrix} = 0 - 36 = -36 < 0$

So $(0,0)$ is a saddle point.

At $(1, 1/2)$:

$\begin{vmatrix} 6 & -6 \\ -6 & 24 \end{vmatrix} = 144 - 36 = 108 > 0$

and $f_{xx} = 6 > 0$ so this is a minimum.

At $(-1, 1/2)$:

$\begin{vmatrix} -6 & -6 \\ -6 & 24 \end{vmatrix} = -144 - 36 = -180 < 0$

so this is a saddle.

2. Find the directional derivative of $f(x, y) = x^2e^{-y}$ at the point $(-2, 0)$ in the direction $< 4, -3 >$.

$$\nabla f = \langle f_x, f_y \rangle = \langle 2xe^{-y}, -x^2e^{-y} \rangle \Big|_{(-2,0)} = \langle -4, -4 \rangle$$

$$|\langle 4, -3 \rangle| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\vec{u} = \frac{\langle 4, -3 \rangle}{5}$$

$$D_u f(-2,0) = \langle -4, -4 \rangle \cdot \frac{\langle 4, -3 \rangle}{5}$$

$$= \frac{-4(4) + -4(-3)}{5}$$

$$= \frac{-16 + 12}{5}$$

$$= -4/5$$

3. Find a direction in which the directional derivative of $f(x, y) = x^2e^{-y}$ at the point $(-2, 0)$ is maximized.

Recall $D_{\vec{u}} f(x)$ is maximized in the direction of the gradient, which maximum value $|\nabla f(x)|$. Above we found

$$\nabla f = \langle 2xe^{-y}, -x^2e^{-y} \rangle \Big|_{(-2,0)} = \langle -4, -4 \rangle$$

So at $(-2,0)$, $D_{\vec{u}} f(x)$ is maximized in the direction $\langle -4, -4 \rangle$, or simpler, the direction $\langle -1, -1 \rangle$.