MAT 397 CAL III Section M005 Spring 2016

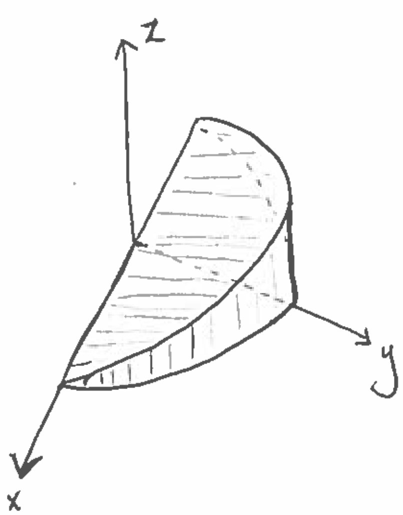
Solutions

Quiz 7

Your Name (please PRINT): Caleb McWhorter

Student ID Number: ______

1. Find the volume of the solid under the plane $y = z$, above the plane $z = 0$ and within the cylinder $x^2 + y^2 = 1$.



$$V = \iiint dV = \int_{-1}^{1}\int_0^{\sqrt{1-x^2}}\int_0^{y} dz\,dy\,dx$$

$$= \int_{-1}^{1}\int_0^{\sqrt{1-x^2}} y\,dy\,dx$$

$$= \frac{1}{2}\int_{-1}^{1} y^2\Big|_0^{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2}\int_{-1}^{1} 1 - x^2\,dx$$

$$= \frac{1}{2}\left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{1}{2}\left[(1 - 1/3) - (-1 + 1/3)\right]$$

$$= 1/2\,(2 - 2/3) = 1 - 1/3 = 2/3$$

OR

$$= \int_0^1 1 - x^2\,dx$$

$$= x - \frac{x^3}{3}\Big|_0^1$$

$$= 2/3$$

OR

$$\int_0^{\pi}\int_0^1\int_0^{r\sin\theta} r\,dz\,dr\,d\theta$$

$$\int_0^{\pi}\int_0^1 r^2\sin\theta\,dr\,d\theta$$

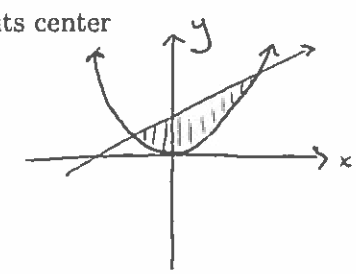
$$\int_0^{\pi}\sin\theta\,d\theta \cdot \int_0^1 r^2\,dr$$

$$-\cos\theta\Big|_0^{\pi} \cdot \frac{r^3}{3}\Big|_0^1$$

$$2 \cdot 1/3$$

$$2/3$$

2. A lamina occupies the region D that is bounded by $y = x^2$ and $y = x + 2$ and the density is $\rho(x, y) = x$. Find the x coordinate of its center of mass.



$$y = y$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x+1)(x-2) = 0$$
$$x = -1 \text{ or } x = 2$$
$$y = 1 \qquad y = 4$$

Total Mass $= \iint_R \rho(x,y)\,dA$

$$= \int_{-1}^{2}\int_{x^2}^{x+2} x\,dy\,dx$$

$$= \int_{-1}^{2} x\,(x + 2 - x^2)\,dx$$

$$= \int_{-1}^{2} x^2 + 2x - x^3\,dx$$

$$= \frac{x^3}{3} + x^2 - \frac{x^4}{4}\Big|_{-1}^{2}$$

$$= \left(\frac{8}{3} + 4 - 4\right) - \left(-1/3 + 1 - \frac{1}{4}\right)$$

$$= 8/3 + 1/3 - 1 + 1/4$$

$$= 3 - 1 + 1/4 = 2\,1/4 = 9/4$$

x Moment: $\iint_R x\,\rho(x,y)\,dA$

$$= \int_{-1}^{2}\int_{x^2}^{x+2} x^2\,dy\,dx$$

$$= \int_{-1}^{2} x^2\,(x + 2 - x^2)\,dx$$

$$= \int_{-1}^{2} x^3 + 2x^2 - x^4\,dx$$

$$= \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{x^5}{5}\Big|_{-1}^{2}$$

$$= \left(4 + \frac{16}{3} - \frac{32}{5}\right) - \left(\frac{1}{4} - \frac{2}{3} + \frac{1}{5}\right)$$

$$= 4 + \frac{16}{3} - \frac{32}{5} - \frac{1}{4} + \frac{2}{3} - \frac{1}{5}$$

$$= \frac{240 + 320 - 384 - 15 + 40 - 12}{60}$$

$$= 189/20 = 63/20$$

$$\bar{x} = \frac{1}{9/4}\cdot\frac{63}{20} = 7/5$$