MAT 397 CAL III Section M005 Spring 2016

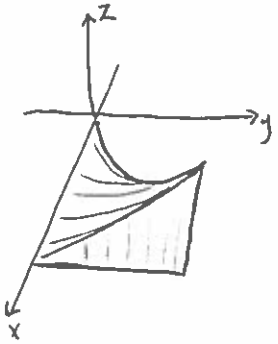
Solutions

Quiz 8

Your Name (please PRINT): Caleb McWhorter

Student ID Number: _____

1. Evaluate the triple integral $\iiint_E 2y\,dV$, where $E = \{(x, y, z) \mid 0 \le y \le 1,\ y \le x \le 1,\ 0 \le z \le xy\}$



$$\iiint_E 2y\,dV = \int_0^1 \int_y^1 \int_0^{xy} 2y\,dz\,dx\,dy$$

$$= \int_0^1 \int_y^1 2xy^2\,dx\,dy$$

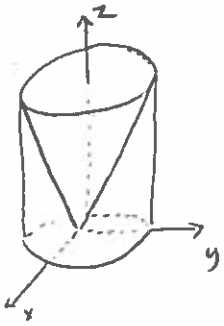
$$= \int_0^1 x^2y^2 \Big|_y^1\,dy$$

$$= \int_0^1 y^2 - y^4\,dy$$

$$\frac{y^3}{3} - \frac{y^5}{5}\Big|_0^1$$

$$\frac{1}{3} - \frac{1}{5} = \frac{5}{15} - \frac{3}{15} = 2/15$$

2. Evaluate $\iiint_E x^2\,dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.



$$\iiint_E x^2\,dV = \int_0^{2\pi} \int_0^1 \int_0^{2r} r^2\cos^2\theta \cdot r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^1 \int_0^{2r} r^3\cos^2\theta\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^1 2r^4\cos^2\theta\,dr\,d\theta$$

$$= \int_0^{2\pi} \cos^2\theta\,d\theta \cdot \int_0^1 2r^4\,dr$$

$$= \frac{1}{2}\int_0^{2\pi} 1 + \cos 2\theta\,d\theta \cdot \int_0^1 2r^4\,dr$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{2\pi} \cdot \frac{2r^5}{5}\Big|_0^1$$

$$= \frac{1}{2}\left[2\pi\right] \cdot \frac{2}{5}$$

$$= \frac{2\pi}{5}$$