MAT 397 CAL III Section M005 Spring 2016

Solution

Quiz 9

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1. Determine whether the vector field $\mathbf{F}(x, y) = e^y\mathbf{i} + (xe^y + \cos y)\mathbf{j}$ is conservative. If yes, find a potential function for $\mathbf{F}$.

$$\vec{F}(x,y) = \langle e^y, xe^y + \cos y, 0 \rangle$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^y & xe^y + \cos y & 0 \end{vmatrix}$$

$$= \hat{\imath}(0-0) - \hat{\jmath}(0-0) + \hat{k}(e^y - e^y)$$

$$= \vec{0}$$

Therefore, $\vec{F}(x,y)$ is conservative. So there is a $f(x,y)$ so that $\vec{F}(x,y) = \nabla f(x,y)$

So $\langle f_x, f_y \rangle = \langle e^y, xe^y + \cos y \rangle$

$$f_x = e^y$$

$$\int f_x \, dx = \int e^y \, dx$$

$$f = xe^y + g(y)$$

Then....

$$\frac{df}{dy} = xe^y + g'(y)$$

but this must be...

$$xe^y + g'(y) = xe^y + \cos y$$

$$g'(y) = \cos y$$

$$\int g'(y)\,dy = \int \cos y \, dy$$

$$g(y) = \sin y + C$$

Then...

$$f(x,y) = xe^y + \sin y + C$$

2. Find the work done by the force field $\mathbf{F}(x,y)$ defined as in the previous problem in moving an object from point $P(0,0)$ to $Q(1,0)$

From (1), we Know $F = \nabla f$, where $f(x,y) = xe^y + \sin y + C$

Then....

$$\text{Work} = \int_C \vec{F} \cdot dr = \int_C \vec{\nabla f} \cdot d\vec{r} = f(1,0) - f(0,0)$$

$$= (1 \cdot e^0 + \sin 0 + C) - (0 + \sin 0 + C)$$

$$= 1 + C - C$$

$$= 1$$

3. Find the arc length of the curve given by $\mathbf{r}(t) =< \frac{t^2}{2}, \frac{t^3}{3} >$, where t is from 0 to 1.

$$r(t) = \langle t^2/2, t^3/3 \rangle$$

$$r'(t) = \langle t, t^2 \rangle$$

$$|r'(t)| = \sqrt{t^2 + t^4}$$

$$L = \int_0^1 |r'(t)| \, dt$$

$$= \int_0^1 \sqrt{t^2 + t^4} \, dt$$

$$= \int_0^1 \sqrt{t^2(1+t^2)} \, dt$$

$$= \int_0^1 t\sqrt{1+t^2} \, dt$$

$$= \frac{1}{3} \int_0^1 3t \sqrt{1+t^2} \, dt$$

$$= \frac{1}{3} \cdot (1+t^2)^{3/2} \Big|_0^1$$

$$= \frac{1}{3}\left[2^{3/2} - 1^{3/2}\right] = \frac{\sqrt{8} - 1}{3}$$