

Name: Caleb McWhorter

Problem 1 (15 points)

- a) Show that the vectors $\langle 1, 3, 2 \rangle$ and $\langle 1, -5, 7 \rangle$ are perpendicular.

Vectors \vec{u}, \vec{v} are $\perp \iff \vec{u} \cdot \vec{v} = 0$

$$\langle 1, 3, 2 \rangle \cdot \langle 1, -5, 7 \rangle = 1(1) + 3(-5) + 2(7) = 1 - 15 + 14 = 0$$

- b) Find $\mathbf{a} \cdot \mathbf{b}$ if $|\mathbf{a}| = 6$, $|\mathbf{b}| = \sqrt{2}$ and if the angle between \mathbf{a} and \mathbf{b} is 45° .

$$\begin{aligned} \vec{a} \cdot \vec{b} &= |\vec{a}| |\vec{b}| \cos \theta \\ &= 6 \cdot \sqrt{2} \cdot \cos 45^\circ \\ &= 6\sqrt{2} \cdot \frac{1}{\sqrt{2}} \\ &= 6 \end{aligned}$$

- c) Find the angle between $\mathbf{a} = \langle 1, 0, -1 \rangle$ and $\mathbf{b} = \langle 0, 1, -1 \rangle$.

We know...

$$\vec{a} \cdot \vec{b} = 1(0) + 0(1) + -1(-1) = 1$$

$$|\vec{a}| = \sqrt{1^2 + (0)^2 + (-1)^2} = \sqrt{2}$$

$$|\vec{b}| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$1 = \sqrt{2} \sqrt{2} \cos \theta$$

$$1 = 2 \cos \theta$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1}(\frac{1}{2})$$

$$\theta = \frac{\pi}{3} \text{ or } 60^\circ$$

Problem 2 (5 points)

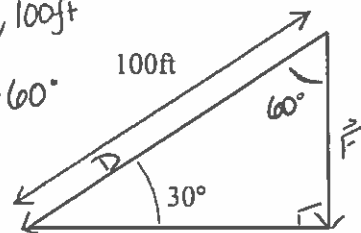
Suppose a sled of mass 18 kg slides down a hill inclined at an angle of 30° . If the length of the hill is 100 m (see figure), what is the work done by the gravitational force?

* If $18 \text{ lb}, 100 \text{ ft}$

1) $18 \cdot 100 \cos 60^\circ$

or
2) $18 \cdot 50$

$20 \sin 30^\circ$
As lb's is a force
g is a mass

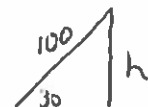


$$g = 9.81 \approx 10$$

$$|F| = mg = 18 \cdot 10 = 180$$

$$\begin{aligned} W &= \vec{F} \cdot \vec{D} \\ &= |\vec{F}| |\vec{D}| \cos \theta \\ &= 180 \cdot 100 \cos 60^\circ \\ &= 180 \cdot 100 \cdot \frac{1}{2} \\ &= 18000 \cdot \frac{1}{2} \\ &= 9000 \text{ J} \end{aligned}$$

OR



$$\begin{aligned} h &= 100 \sin 30^\circ \\ &= 100 \cdot \frac{1}{2} \\ &= 50 \end{aligned}$$

$$\begin{aligned} W &= Fd \\ &= 180 \cdot 50 \\ &= 9000 \text{ J} \end{aligned}$$