Solutions

MAT 397, Section 001, Spring 2016 **Quiz 4, February 22**

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Problem 1 (10 points)

Compute the length of the curve $\mathbf{r}(t) = \langle 3\cos t, 4t, 3\sin t\rangle$ where $0 \le t \le 2$.

$\vec{r}'(t) = \langle -3\sin t, 4, 3\cos t\rangle$

$|\vec{r}'(t)| = \sqrt{(-3\sin t)^2 + 4^2 + (3\cos t)^2}$

$= \sqrt{9\sin^2 t + 9\cos^2 t + 16}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

$L = \int_0^2 |r'(t)|\, dt$

$= \int_0^2 5\, dt$

$= 10$

Problem 2 (10 points)

If the acceleration of a particle at time t is given by $\mathbf{a}(t) = \langle e^t, 2t, 3t^2\rangle$ and the velocity of the particle at time $t = 0$ is the zero vector, what is the velocity of the particle at time $t = 2$?

$\vec{a}(t) = \langle e^t, 2t, 3t^2\rangle$

$\vec{v}(0) = \vec{0}$

$\vec{v}(t) = \int \vec{a}(t)\, dt$

$= \int \langle e^t, 2t, 3t^2\rangle\, dt$

$= \langle e^t, t^2, t^3\rangle + \langle C_1, C_2, C_3\rangle$

$\vec{v}(0) = \langle 0,0,0\rangle = \langle e^0 + C_1, 0^2 + C_2, 0^3 + C_3\rangle$

$1 + C_1 = 0$

$C_2 = 0$

$C_3 = 0$

$\therefore C_1 = -1$

$\vec{v}(t) = \langle e^t - 1, t^2, t^3\rangle$

$\vec{v}(2) = \langle e^2 - 1, 4, 8\rangle$