MAT 397, Section 001, Spring 2016 Solutions **Quiz 6, March 7**

Name: Caleb McWhorter

Problem 1 (10 points) Let $f(x,y) = 3xy + x^4 + y^3 + 1$.

a) Compute the gradient vector of f at the point (1,2).

$\nabla f = \langle f_x, f_y \rangle = \langle 3y + 4x^3, 3x + 3y^2 \rangle$

$\nabla f(1,2) = \langle 3(2) + 4(1)^3, 3(1) + 3(2)^2 \rangle$

$= \langle 6+4, 3+12 \rangle$

$= \langle 10, 15 \rangle$

b) Compute the directional derivative of f at the point (1,2) in the direction of $\mathbf{u} = \langle 3/5, 4/5 \rangle$.

$|\vec{u}| = \frac{1}{5}\sqrt{3^2+4^2} = \frac{1}{5}\sqrt{25} = 1$ $\quad = \frac{1}{5}\langle 3,4 \rangle$

$D_{\vec{u}} f(1,2) = \nabla f(1,2) \cdot \vec{u} = \langle 10, 15 \rangle \cdot \langle 3/5, 4/5 \rangle$

$= 10(3/5) + 15(4/5)$

$= 6 + 12$

$= 18$

Problem 2 (10 points) Find the critical points of $f(x,y) = 2x^2 - 4xy + y^4$.

$\begin{cases} f_x = 4x - 4y = 0 \\ f_y = -4x + 4y^3 = 0 \end{cases}$

Now...

$4x - 4y = 0$

$4(x-y) = 0$

$x - y = 0$

$x = y$

So...

$-4x + 4y^3 = 0$

$-4y + 4y^3 = 0$

$y(-4 + 4y^2) = 0$

$y(4y^2 - 4) = 0$

$y(2y-2)(2y+2) = 0$

$y = 0 \quad 2y - 2 = 0 \quad 2y + 2 = 0$

$y = 1 \quad y = -1$

$x = 0 \quad x = 1 \quad x = -1$

Then the critical points are...

$(0,0)$

$(1,1)$

$(-1,-1)$