

Name:

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Solutions

Problem 1 (10 points) Use cylindrical coordinates to find the volume of the solid $E =$

$$\{(x, y, z) \mid 1 \leq \sqrt{x^2 + y^2} \leq 2, 0 \leq z \leq \frac{1}{\sqrt{x^2 + y^2}}\}.$$

$$1 \leq \sqrt{x^2 + y^2} \leq 2$$

$$1 \leq x^2 + y^2 \leq 4$$

$$0 \leq z \leq \frac{1}{\sqrt{x^2 + y^2}}$$

$$0 \leq z \leq 1/r$$

$$V = \iiint_E dV = \int_0^{2\pi} \int_1^2 \int_0^{1/r} r \, dz \, dr \, d\theta$$

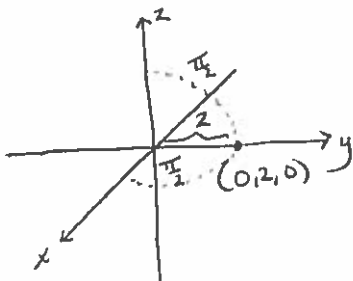
$$= \int_0^{2\pi} \int_1^2 r \cdot z \Big|_0^{1/r} \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_1^2 \, dr \, d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \int_1^2 \, dr$$

$$= 2\pi \cdot 1$$

$$= 2\pi$$

Problem 2 (10 points) The point $(\rho, \theta, \phi) = (2, \pi/2, \pi/2)$ is given in spherical coordinates. Plot the point and find its rectangular coordinates.

$$x = 2 \sin \frac{\pi}{2} \cos \frac{\pi}{2} = 0$$

$$y = 2 \sin \frac{\pi}{2} \sin \frac{\pi}{2} = 2$$

$$z = 2 \cos \frac{\pi}{2} = 0$$