

Math 296: Exam 1
Summer Session II – 2016
07/14/2016
80 Minutes

Name: _____

Write your name on the appropriate line on the exam cover sheet. This exam contains 9 pages (including this cover page) and 8 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
Total:	40	

1. (5 points) Find the limits of the following *sequences*:

$$a_n = \frac{3n^2 - 5n + 7}{5n^2 - 2n + 3}$$

$$b_n = n \sin\left(\frac{1}{n}\right)$$

$$c_n = \sqrt[n]{2n}$$

$$d_n = \left(1 + \frac{7}{5n}\right)^{2n/3}$$

2. (5 points) Determine if the following series converge or diverge. Justify your answer.

$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$$

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

3. (5 points) Determine if the following series converges or diverges. If the series converges, find the sum. If the series diverges, prove it. [Hint: $\frac{1}{n^2 + 3n + 2} = \frac{1}{n + 1} - \frac{1}{n + 2}$]

$$\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$$

4. (5 points) Determine if the following series converges or diverges. If the series converges, find the sum. If the series diverges, prove it.

$$\sum_{n=0}^{\infty} \frac{3^{n-2}}{e^{n+3}}$$

$$\sum_{n=1}^{\infty} \frac{\pi^n - 2^{n+1}}{3^{2n}}$$

5. (5 points) Determine if the following series conditionally converges, absolutely converges, or diverges. If the series converges, determine how many terms are needed to add to approximate the sum to three decimal digits of accuracy. If the series diverges, prove it.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$$

6. (5 points) Determine if the following series converge or diverge. Be sure to justify your answer.

$$\sum_{n=1}^{\infty} \frac{n^2 + n}{n^4 + 5}$$

$$\sum_{n=1}^{\infty} \frac{n + \ln n}{n^2 + 1}$$

7. (5 points) Determine if the following series converge or diverge. Be sure to be as specific as possible and justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-5)^n}{3^n n^n}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n \sqrt{n}}{n!}$$

8. (5 points) Complete the following question by completing each part. Be sure to justify your answer for each part.

(a) Find the first three nonzero terms of the Maclaurin series for $\frac{1}{1-x}$.

(b) Use the previous part to find the Maclaurin series for $\frac{1}{1-x}$.

(c) Find the interval and radius of convergence of the power series in (b).

(d) Use the work in the previous parts to find the sum $\sum_{n=0}^{\infty} \frac{n}{3^n}$.