Math 296: Final Exam **Name:** _____
Summer Session II – 2016
08/11/2016
145 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 25 pages (including this cover page) and 24 questions. Check that you have every page of the exam.

Do seven questions from Problems 1 to 9, six questions from Problems 10 to 17, and five questions from Problems 18 to 24 – for a total of eighteen problems. Clearly indicate which problems you wish to be graded by placing a checkmark in the circle at the top of the page. Only questions which have a checkmark will be graded.

For each question, write your answer in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page – being sure to indicate the problem number. Each question is out of 5 points.

Problem	Score	Problem	Score	Problem	Score
1		10		18	
2		11		19	
3		12		20	
4		13		21	
5		14		22	
6		15		23	
7		16		24	
8		17			
9					
Total		Total		Total	

Exam Total	Average	Percentage

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1. (5 points) Find the limits, if they exist, of the following sequences:

$$\lim_{n\to\infty} \cos(1/n) + \sqrt[n]{2n}$$

$$\lim_{n\to\infty} \frac{2n^2 - 3n + 7}{3n^2 + 2n + 5}$$

$$\lim_{n\to\infty} \ln(2n + 3) - \ln(3n - 5)$$

$$\lim_{n\to\infty} \sin\left(\frac{n\pi}{2}\right)$$

$$\lim_{n\to\infty} \left(1 + \frac{2}{3n}\right)^{3n/5}$$

2. (5 points) Determine whether the following series converge or diverge. Be sure to justify your answer.

$$\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n}\right)$$

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

3. (5 points) Determine if the following series converge or diverge. If the series converges, find the sum. If the series diverges, explain why.

$$\sum_{n=1}^{\infty} \frac{5\pi^{n+1}}{3^{n-1}}$$

$$\sum_{n=1}^{\infty} \frac{3^n + 1}{2^{2n}}$$

4. (5 points) Determine if the following series converges or diverges. If the series converges, find the sum. If the series diverges, explain why.

$$\sum_{n=0}^{\infty} \frac{2}{n^2 + 4n + 3}$$

5. (5 points) Determine if the following series converge or diverge. Be sure to justify your answers.

$$\sum_{n=1}^{\infty} \frac{2n-5}{\sqrt{3n^6-2}}$$

$$\sum_{n=1}^{\infty} \frac{7+4\sqrt[3]{n^4}}{n^2+2n+1}$$

6. (5 points) Determine if the following series converges or diverges. If the series converges, determine at most how many terms are needed to approximate the sum to 4 decimal digits of accuracy. If the series diverges, explain why.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$$

7. (5 points) Determine if the following series converge or diverge. Be sure to justify your answer.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

$$\sum_{n=1}^{\infty} \left(\frac{2n^3 + 5n^2 - n + 6}{3n^3 + 6n^2 - n + 7} \right)^n$$

8. (5 points) Determine the interval and radius of convergence of the following power series:

$$\sum_{n=1}^{\infty}(-1)^n \frac{(x+2)^n}{n\,2^n}$$

9. (5 points) Use known Taylor/Maclaurin Series to evaluate the following:

$$\int x \cos x^3 \, dx$$

$$\sum_{n=0}^{\infty} \frac{n}{5^n}$$

10. (5 points) Integrate the following:

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$$

$$\int \frac{e^{2x}}{e^{2x} + 1}\, dx$$

11. (5 points) A table of integrals gives the following:

$$\int \frac{dx}{x^3(x^4 - a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6} \ln\left(\frac{x^2 - a^2}{x^2 + a^2}\right)$$

Use this to evaluate the following:

$$\int \frac{e^{-2x}}{e^{4x} - 1}\, dx$$

12. (5 points) Evaluate the following integrals:

$$\int x^4 \ln x \, dx$$

$$\int \frac{x^5}{\sqrt{1-x^3}} \, dx$$

13. (5 points) Evaluate the following integral:

$$\int x^4 \sin(2x)\, dx$$

14. (5 points) Evaluate the following integral:

$$\int e^x \sin 2x \; dx$$

15. (5 points) Evaluate the following integral:

$$\int \frac{3x^2 + x - 2}{x^3 - x^2 + x - 1}\, dx$$

16. (5 points) Evaluate the following integral:

$$\int \sin^2 x \, \cos^3 x \, dx$$

17. (5 points) Integrate the following:

$$\int \frac{\sqrt{x^2-4}}{x^3}\,dx$$

18. (5 points) Evaluate the following integral:

$$\int_1^\infty \frac{dx}{2x^2 + x}$$

19. (5 points) Complete the following parts:

(a) Convert the polar coordinate $(2, 5\pi/3)$ to cartesian coordinates.

(b) Convert the cartesian coordinate $(-1, \sqrt{3})$ to polar coordinates.

(c) Find the equation of the tangent line of $r(\theta) = 1 + \sin\theta$ at $\theta = 0$.

20. (5 points) Set up the integrals to calculate—but do not evaluate—the following:

(a) The area between the curves $y = 2\sqrt[4]{x}$ and $y = 2x^5$.

(b) The volume revolving the region bound by the curves $y = 2\sqrt[4]{x}$ and $y = 2x^5$ about the line $x = 5$. You must set-up the integrals using both the Disk/Washer Method and the Shell Method.

21. (5 points) The base of a solid is given by the region bound by $y = x^2 - 2x$ and $y = x + 4$. Cross sections of this solid perpendicular to the x-axis are isosceles right triangles with hypotenuse lying in the plane. Set up the integral to calculate—but do not evaluate—the volume of the solid.

22. (5 points) Set up the integrals to calculate—but do not evaluate—the following:

(a) The length of the curve $y^2 - 4x = 0$ from $(1, 2)$ to the point $(4, 4)$.

(b) The surface area generated by revolving the section of the curve $y^2 - 4x = 0$ from $(1, 2)$ to $(4, 4)$ around the line $y = -2$.

23. (5 points) Set up the integrals to calculate—but do not evaluate—the following:

(a) The work to lift a 150 kg load 100 m using a steel chain that weighs 5 kg/m.

(b) The work to pump out a liquid with density 900kg/m^3 out of a conical tank, set up so that it is resting on a point, of height 10m with radius 3m that is filled to a depth of 8m.

24. (5 points) Solve the following differential equations:

$x^2y' + 3xy = 1$

$y' - xe^y = 0$