Problem 1: Integral Test

Explain why the Integral Test applies to the following series. Then by integrating (or using WolframAlpha), show that the series converges/diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}-10}{n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+n^2}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$

Problem 2: Comparison Test

Use the Comparison Test to determine if the following series converge/diverge:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n-7}{6+5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n}{n^3-4}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\ln n}{n^3+1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-3}{\sqrt{3n^3-n+1}}$

Problem 3: Limit Comparison Test

Use the Limit Comparison Test to determine if the following series converge/diverge:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 \ln n}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+3}{\sqrt{n^5+2}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3+7}}{n^2+2n+3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

Problem 4: Root Test

Use the Root Test to determine whether the following series converge/diverge:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{5^n(n^3+2)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan^n n \ln n}{\sqrt[3]{n^5}}$

Problem 5: Ratio Test

Use the Ratio Test to determine whether the following series converge/diverge:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{2^n n!}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(n^2+1)2^n}{n!\sqrt{n-1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3n+2}{3^n+5}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(2n)!}{n^n}$

Problem 6: Alternating Series

Use the Alternating Series Test to determine whether the following series converge/diverge. Be sure to check if the converge conditionally or absolutely. If the series converges, find the sum of the first 3 terms (of the alternating series) and explain at most how far this number is from the actual sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{e^n}$

Problem 7: Mix 'n Match

In each of the following series, determine if the series converges or diverges. For each of the series, use as many possible series tests as possible.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+n+2}{n^4-3n^2+2n+1}$

(b) $\displaystyle\sum_{n=5}^{\infty} \frac{2n}{\sqrt{3n^5-2n-3}}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{\pi^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n+3}$