

***u*-substitution**

Integrate the following:

(a) $\int \frac{x^2}{x-1} dx$

(b) $\int \frac{2^x}{2^x+1} dx$

(c) $\int x^2 \sqrt[3]{x^3-1} dx$

(d) $\int e^{\sin x} \cos x dx$

(e) $\int \left(\frac{3x-1}{5}\right) dx$

Table of Integrals

The following can be found on a table of integrals:

$$\int \frac{dx}{x^3(x^4-a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6} \ln \left(\frac{x^2-a^2}{x^2+a^2} \right)$$
$$\int \frac{dx}{x\sqrt{x^n-a^n}} = \frac{2}{n\sqrt{a^n}} \cos^{-1} \sqrt{\frac{a^n}{x^n}}$$
$$\int \frac{dx}{1-\cos ax} = \frac{-1}{a} \cot \left(\frac{ax}{2} \right)$$
$$\int x \cot^{-1} \left(\frac{x}{a} \right) dx = \frac{(x^2+a^2)}{2} \cot^{-1} \left(\frac{x}{a} \right) + \frac{ax}{2}$$

Use these to integrate the following:

(a) $\int \frac{\ln 2}{\sqrt{2^{5x}-32}} dx$

(b) $\int \frac{dx}{\sec x(1-\cos(2\sin x))}$

(c) $\int \frac{e^{-2x}}{e^{4x}-1} dx$

(d) $\int \frac{\ln 2 \operatorname{arccot} (2^{x-1})}{2^{-2x}} dx$

Trigonometric Integration

Integrate the following:

(a) $\int \sin^2 5x \, dx$

(b) $\int \frac{\tan^3 x}{\sqrt{\sec x}} \, dx$

(c) $\int \sec x \tan^3 x \, dx$

(d) $\int \sin^5 x \cos^2 x \, dx$

Partial Fractions

Integrate the following using partial fractions. You may only check your answers using Heaviside's Method; you may not use it to directly find values.

(a) $\int \frac{2x - 7}{x^2 - 7x + 12} \, dx$

(b) $\int \frac{x^2 + 3x - 4}{(x + 1)^2(x - 5)} \, dx$

(c) $\int \frac{x^2 + x + 3}{x^3 + x^2 + 2x + 2} \, dx$

(d) $\int \frac{-x^3 + 15x^2 + 3x + 13}{(x^2 - x - 6)(x^2 + x + 1)} \, dx$

(e) $\int \frac{3x^4 - 2x^3 + 13x^2 - 6x + 15}{x^2 + 3} \, dx$

Trig Substitution

Integrate the following:

(a) $\int \frac{dx}{\sqrt{16 - x^2}}$

(b) $\int \frac{dx}{\sqrt{(x^2 + 5)^3}}$

(c) $\int \frac{3}{x^2 \sqrt{9 - x^2}} \, dx$

(d) $\int \frac{x^2}{(1 + x^2)^2} \, dx$

(e) $\int \frac{x^3}{\sqrt{x^2 - 4}} \, dx$