Problem 1: Cartesian to Polar

Convert the following cartesian coordinates to polar coordinates. Be sure to draw a diagram to illustrate the conversion.

(i) $(3/\sqrt{2}, 3\sqrt{2})$

(ii) $(1/4, -\sqrt{3}/4)$

(iii) $(-1, \sqrt{3})$

(iv) $(-2, -2\sqrt{3})$

(v) $(1/2, \sqrt{3}/4)$

(vi) $(-1/2\sqrt{2}, -1/2\sqrt{2})$

(vii) $(3\sqrt{2}, -3\sqrt{2})$

(viii) $(-3\sqrt{3}/2, 3/2)$

Problem 2: Polar to Cartesian

Convert the following polar coordinates to cartesian coordinates. Be sure to draw a diagram to illustrate the conversion.

(i) $(1, -\pi/6)$

(ii) $(1/2, 7\pi/6)$

(iii) $(4, 11\pi/6)$

(iv) $(2, 3\pi/4)$

(v) $(3, 2\pi/3)$

(vi) $(1, 7\pi/4)$

(vii) $(1/4, \pi/6)$

(viii) $(2, -11\pi/6)$

Problem 3: Cartesian Plots to Polar

Convert the following equations to polar form:

(i) $y = 5x - 2$

(ii) $xy^2 = 1$

(iii) $x^2 + y^2 = 4$

(iv) $x^2 + 3y^2 = 5$

(v) $x = 2$

(vi) $y = -4$

(vii) $y = 2 - x^2$

Problem 4: Polar Plots

Plot the following polar functions:

(i) $r = \theta$

(ii) $r = 2\cos 4\theta$

(iii) $r^2 = 2\cos 2\theta$

(iv) $r = 2\sin 4\theta$

(v) $r^2 = 1/\theta$

(vi) $r = 5 - 5\sin\theta$

(vii) $r = 3 + \sin\theta$

(viii) $r = 5 - 5\cos\theta$

(ix) $r = 1 + 3\cos\theta$

Problem 5: Derivatives with Polar Coordinates

(i) Find the slope of the tangent line for $r(\theta) = 1 + \cos\theta$ at any given θ.

(ii) Find the slope of the tangent line for $r(\theta) = \cos 2\theta$ at any given θ.

(iii) Find the slope of the tangent line for $r(\theta) = \sin^2\theta$ at any given θ.

(iv) Find the slope of the tangent line for $r(\theta) = 1/\theta$ at any given θ.

(v) Find the slope of the tangent line for $r(\theta) = \sin\theta - \cos\theta$ at any given θ.

(vi) Find the second derivative of $r(\theta) = \sin\theta$ at any given θ.

(vii) Find the second derivative of $r(\theta) = \cos\theta$ at any given θ.

(viii) Find the second derivative of $r(\theta) = 1 + \cos\theta$ at any given θ.

(ix) Find the second derivative of $r(\theta) = 1/\theta$ at any given θ.

Problem 6: Area with Polar Coordinates

(i) Find the area inside the cardioid $r(\theta) = 1 + \cos\theta$.

(ii) Find the area between $r(\theta) = 2$ and $r(\theta) = 3\sin\theta$.

(iii) Find the area inside of $r = \sin^2\theta$.

(iv) Find the area inside of $r = \cos 3\theta$.

(v) Find the area inside of the inner loop of $r(\theta) = 2 + 4\cos\theta$.

(vi) Find the area inside $r(\theta) = 2 + 2\sin\theta$ and outside $r(\theta) = 2$.

(vii) Find the area outside of $r(\theta) = 5 + 3\sin\theta$ and inside $r(\theta) = 3$.

Problem 7: Area between Curves

Find the area between the given curves:

(i) $f(x) = x^2$, $g(x) = 0$, $x = -2$, $x = 2$

(ii) $f(x) = x$, $g(x) = \sin x$

(iii) $f(x) = x^2 - 10$, $g(x) = 5 - 2x$

(iv) $f(x) = \sin x$, $g(x) = \cos x$, $x = 0$, $x = 2\pi$

(v) $f(x) = \sqrt{x}$, $g(x) = x$

(vi) $f(x) = x^2$, $g(x) = \sqrt[3]{x}$

(vii) $f(x) = x^3 - 10x + 3$, $g(x) = 3 - 3x^2$

(viii) $f(x) = x^2$, $y = 4$, $x = 0$

(ix) $f(x) = \sqrt[5]{x}$, $x = 0$, $y = 32$

(x) $y = x - 1$, $y^2 = 2x + 6$

(xi) $x = y^2 - 4$, $x = y + 2$

(xii) $x = y^3 - 10y + 3$, $x = 3 - 3y^2$

Problem 8: Volume of Rotations

Set up the integral for calculating – but do not compute – the volume of rotating the given region about the given axis. Whenever possible, set up the integral for both the Shell Method and the Disk/Washer Method.

(i) $y = 2x - 4$, $x = 6$, $y = 0$ about the x-axis

(ii) $y = 2x - 4$, $x = 6$, $y = 0$ about the y-axis

(iii) $y = x^2$, $y = 3$, $x = 0$ about the x-axis

(iv) $y = x^2$, $y = 3$, $x = 0$, about the y-axis

(v) $y = \sqrt[3]{x}$, $y = x/4$, about the x-axis

(vi) $y = \sqrt[3]{x}$, $y = x/4$, about the line $x = 6$

(vii) $y = \sqrt[3]{x}$, $y = x/4$, about the line $x = -5$

(viii) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $y = 7$

(ix) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $y = -5$

(x) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $x = 7$

(xi) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $x = -4$

Problem 9: Area by Cross Sections

(i) The base of a solid has boundary given by the curves $f(x) = x^2 - 1$ and $g(x) = 1 - x^2$. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

(ii) The base of a solid has boundary given by the curves $f(x) = x^2 - 1$ and $g(x) = 1 - x^2$. The cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

(iii) Find the volume of a solid pyramid with square base that is 5 units tall and 20 units on the side.

(iv) A regular cone has a base that is 4 units across and 5 units tall. Find the volume of the cone.

(v) The base of a solid has boundary given by $y = 4 - x^2/9$ and $y = 0$. Cross sections perpendicular to the x-axis are rectangles with heights twice the length of the side lying in the plane. Find the volume of this solid.

(vi) The base of a solid has boundary given by $y = \sqrt{4 - x^2}$ and $y = 0$. Cross sections parallel to the x-axis are squares. Find the volume of the solid.

(vii) The base of a solid has boundary given by the ellipse $4x^2 + 9y^2 = 9$. Cross sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse lying in the plane. Find the volume of the solid.

(viii) The base of a solid has boundary given by $x^2 + y^2 = 4$. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

(ix) The base of a solid is given by the curve $y = \sin x$ from 0 to π and the curve $y = 0$. Cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

(x) The base of a solid is given by the curves $y = \sqrt{x}$ and $y = x^2$. Slices perpendicular to the y-axis are rectangles with height a third the length of the side lying in the plane. Find the volume of the solid.

Problem 10: Arc Length

Set up the integrals to calculate – but do not evaluate – the following lengths:

(i) The length of the curve $y^2 = x$ from $(0, 0)$ to $(1, 1)$

(ii) The length of the curve $y = \sin x$ from $(0, 0)$ to $(2\pi, 0)$

(iii) The length of the curve $xy = 1$ from $(1, 1)$ to $(2, 1/2)$

(iv) The length of the curve $x^2 + y^2 = 9$ from $(3, 0)$ to $(0, -3)$

(v) The length of the curve $y^2 = 4(x + 4)^3$ for $x \in [0, 2]$ and $y > 0$.

(vi) The length of the curve $x = y^2 + 3$ from $(4, -1)$ to $(4, 1)$

(vii) The length of the curve

$$\begin{cases} x(t) = t^3 + 2t + 1 \\ y(t) = 2t + 1 \end{cases}$$

from $t = 0$ to $t = 1$

(viii) The length of the curve

$$\begin{cases} x(t) = 2\cos t \\ y(t) = 3\sin t \end{cases}$$

for $t = 0$ to $t = 2\pi$.

(ix) The length of the curve

$$\begin{cases} x(t) = e^{-t} \\ y(t) = 2t\sin t \end{cases}$$

for $t = 0$ to $t = 3\pi$.

Problem 11: Surface Area

Set up the integrals to find – but do not evaluate – the following:

(i) The surface area from rotating $y = \sqrt{9 - x^2}$ for $y \geq 0$ about the x-axis.

(ii) The surface area from rotating $y = \sqrt[3]{x}$ from $(0, 0)$ to $(8, 2)$ about the x-axis.

(iii) The surface area from rotating $y = \sqrt[3]{x}$ from $(0, 0)$ to $(8, 2)$ about the y-axis.

(iv) The surface area from rotating $y = \sqrt{4x + 1}$ from $(0, 1)$ to $(2, 3)$ about the line $x = 6$.

(v) The surface area from rotating $y = \sqrt{4x + 1}$ from $(0, 1)$ to $(2, 3)$ about the line $y = 5$.

(vi) The surface area from rotating $x = \log_7(2y + 1)$ from $(0, 0)$ to $(1, 3)$ about the line $x = -3$.

(vii) The surface area from rotating $x = \log_7(2y + 1)$ from $(0, 0)$ to $(1, 3)$ about the line $y = -3$.

(viii) The surface area from rotating $y = x^3$ from $(0, 0)$ to $(2, 8)$ about the line $y = -2$.

(ix) The surface area from rotating $y = x^3$ from $(0, 0)$ to $(2, 8)$ about the line $x = 6$.

(x) The surface area from rotating $y = x^3$ from $(0, 0)$ to $(2, 8)$ about the line $x = -2$.

(xi) The surface area from rotating $x = 3t^2$, $y = 2t^3$ for $0 \leq t \leq 5$ about the y-axis.