Problem 1: Verifying Solutions

Check if the given functions are solutions to the given differential equations:

(i) $y(t) = e^t$; $y' - 3ty = 0$

(ii) $y(t) = t^2$; $t^2y'' - 2y = 0$

(iii) $y(t) = 1/t$; $t^2y'' - 2y = 0$

(iv) $y(t) = t\sin t$; $t^2y'' - 2y = 0$

(v) $y(t) = e^{3t} + 1$; $y' = 3y$

(vi) $y(t) = 1$; $yy'' + (y')^2 = 0$

(vii) $y(t) = 3 - e^{-t/3}$; $2y' + y = 3$

(viii) $y(t) = \sqrt{t}$; $yy'' + (y')^2 = 0$

(ix) $y(t) = \sin 2t$; $y'' + 4y = 0$

(x) $y(t) = \sqrt{t}$; $y'' - y/t + 3 = 0$

(xi) $y(t) = Ct^3$; $ty' - 3y = 0$

Problem 2: Slope Fields

Plot the following slope/vector fields:

(i) $y' = 2$

(ii) $y' = x$

(iii) $y' = y$

(iv) $y' = x - y$

(v) $y' = x + y$

(vi) $y' = x^2$

(vii) $y' = \cos x$

(viii) $y' = 1/x$

(ix) $y' = x/y$

Problem 3: Separable Equations

Solve the following differential equations:

(i) $xy' = y$

(ii) $y' = 1$

(iii) $y' \sin x = y \ln y$

(iv) $y' = xy + 2x + y + 2$

(v) $y' - xy = x$

(vi) $y' = y^2/(x^2 + 3x + 2)$

(vii) $y' = xe^y$

(viii) $(x + xy)y' + y = 0$

(ix) $y' = x/(3 - y)$

(x) $xy' - xy = y$

Problem 4: Euler's Method

(i) Approximate $y(1)$ if $y' = y - x$ and $y(0) = 1/2$. Use $h = 0.5$.

(ii) Approximate $y(1.2)$ if $y' = x^2 - y$ and $y(0) = 1$. Use $h = 0.4$.

(iii) Approximate $y(0.9)$ if $y' = xy$ and $y(0) = 2$. Use $h = 0.3$.

Problem 5: Homogeneous Equations

Solve the following differential equations:

(i) $(x - 2y)\,dx + x\,dy = 0$

(ii) $x^2\,dy + (y^2 - xy)\,dx = 0$

(iii) $(x - y)\,dx + x\,dy = 0$

(iv) $(x^2 - y^2)\,dx + 2xy\,dx = 0$

(v) $(x^2y + 2xy^2 - y^3)\,dx - (2y^3 - xy^2 + x^3)\,dy = 0$

(vi) $(x \sin(y/x) - y \cos(y/x))\,dx + x \cos(y/x)\,dy = 0$

Problem 6: First-Order Linear Equations

Solve the following differential equations:

(i) $y' + y = e^x$

(ii) $y' + y/x = x^2$

(iii) $y' + y\cos x = \sin 2x$

(iv) $xy' = x^2 + 3y$

(v) $y' + 2xy = 0$

(vi) $x^2y' + 3xy = 1$

(vii) $y' + 4xy = x$

(viii) $(x\ln x)y' + y = \ln x$

(ix) $y' - x = xy$

Problem 7: Applied Integrals

(i) A spring has rest length of 1m. A force of 25N stretches the spring by 0.5m. Determine how much work is done stretching the spring 1.5m beyond its rest length. Determine how much work is done compressing the spring by 0.2m.

(ii) Determine the amount of work required to lift a 5kg block using a thin near massless wire 5m vertically off of the ground.

(iii) A 10kg solid block is attached by a thin near massless wire to a pulley of radius 1m high above the block. Determine the work done lifting the block 6 complete rotations of the pulley.

(iv) A 15kg block sits at the bottom of a well. A rope is used to haul the block to the surface – a distance of 40m. If the rope weights 0.2kg/m, determine the work done in performing this task.

(v) A cylindrical tank of radius 3m and 8m high is half full of water. Determine the amount of work it takes to pump the remaining water out of the tank through a hole in the top of the tank.

(vi) A semicircular tank lies of radius 10m balanced on its “point" on the ground – full of water. Determine the work required to pump the liquid out of the tank if the liquid has a density of 5kg/m^3.

(vii) A triangular trough is filled with water. The trough is 8m long and has a top base shaped as a rectangle of width 3m. The trough is also 2m deep. Assuming the trough is full, determine the amount of work required to empty the first meter of liquid.

(viii) A triangular plate with base 6m in length is submerged vertically so its highest point is deepest in the water – a total depth of 4m. Determine the hydrostatic pressure on the plate.

(ix) Find the hydrostatic force on a circular plate of radius 2m submerged vertically to a depth of 10m.