Name: *Caleb McWhorter — Solutions* **Quiz 11: Series Tests**
MAT 296 **Fall 2017**

Problem 1: Determine whether the following series converges or diverges. Be sure you prove your answer.

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

$$\lim_{n\to\infty} \frac{\sin(1/n^2)}{1/n^2} = 1$$

Now $\sum_{n=1}^{\infty} \frac{1}{n^2}$ *converges by the p-test. Therefore,* $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ *converges by the Limit Comparison Test.*

Problem 2: Determine whether the following series converges or diverges. Be sure you prove your answer.

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n-1}$$

For sufficiently large n,

$$\frac{2n+1}{n^2+n-1} \approx \frac{2n}{n^2} = \frac{2}{n}$$

So we suspect the series diverges.

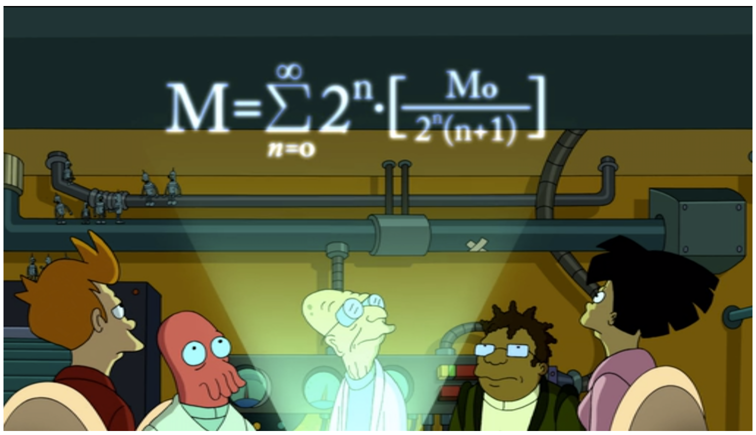
$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n-1} \geq \sum_{n=1}^{\infty} \frac{2n}{n^2+n} \geq \sum_{n=1}^{\infty} \frac{2n}{n^2+n^2} = \sum_{n=1}^{\infty} \frac{1}{n}$$

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ *diverges by the p-test. Therefore,* $\sum_{n=1}^{\infty} \frac{2n+1}{n^2+n-1}$ *diverges by the Comparison Test.*

Bonus: In the episode "Benderama" of the TV show Futurama™, the robot Bender avoids doing work by consuming some mass and making two smaller copies of himself, each 60% of his size, and makes the new copies do the work. Being equally lazy, the copies make copies of themselves to do their work and so forth. At one point in the episode, Professor Farnsworth runs into the room and announces that if this keeps up the total mass of Benders in the world will be given by

$$M = \sum_{n=0}^{\infty} 2^n \cdot \left[\frac{M_0}{2^n(n+1)} \right]$$

where M is the total mass of the Benders ansd M_0 is just some constant representing whatever the *original* Bender weighed. Everyone gasps in horror (except for Fry who does not understand). Explain why everyone is so worried. [Fun Fact: One of the creators/writers of the show has a PhD. in Math, which is why there are so many Math/Science jokes/references throughout the show. Fun Fact 2: The equation Professor Farnsworth gives and what he says are incorrect. *J'accuse!*]



$$M = \sum_{n=0}^{\infty} 2^n \cdot \left[\frac{M_0}{2^n(n+1)} \right] = \sum_{n=0}^{\infty} \frac{M_0}{n+1} = M_0 \sum_{n=0}^{\infty} \frac{1}{n+1} = M_0 \sum_{n=1}^{\infty} \frac{1}{n}$$

This series clearly diverges by the p-test. But then the total mass, M, of Benders becomes infinitely large.

For this to happen, he would have to consume infinite mass. Hence in his pursuit of laziness, the Bender would consume the Earth.