Name: *Caleb McWhorter — Solutions* **Quiz 12: Series Tests**

MAT 296 **Fall 2017**

Problem 1: Determine whether the following series converges or diverges. Be sure to prove your answer.

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$$

The series is alternating. Observe $\lim_{n\to\infty} \frac{1}{\ln n} = 0$. *Clearly,* $\frac{1}{\ln n}$ *is decreasing in n. Therefore,* $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$ *converges by the Alternating Series Test.*

Problem 2: Determine whether the following series converges conditionally, converges absolutely, or diverges. Be sure to justify your answer completely.

$$\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2+4}$$

The series clearly alternates. Observe $\lim_{n\to\infty} \frac{n}{n^2+4} = 0$ *and for sufficiently large n, the series is decreasing:* $\frac{d}{dx}\left(\frac{x}{x^2+4}\right) = \frac{4-x^2}{(x^2+4)^2}$ *so that the sequence is decreasing for* $n > 4$. *[The first 5 terms to not affect the convergence/divergence.] Therefore,* $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2+4}$ *converges by the Alternating Series Test. Now consider the series (noting we ignore the* $n = 0$ *term as it does not effect convergence)*

$$\sum_{n=1}^{\infty} \frac{n}{n^2+4} \geq \sum_{n=1}^{\infty} \frac{n}{n^2+4n^2} = \frac{1}{5}\sum_{n=1}^{\infty} \frac{1}{n}$$

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ *diverges by the p-test. Therefore,* $\sum_{n=0}^{\infty} \frac{n}{n^2+4}$ *diverges by the Comparison Test. Therefore,* $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2+4}$ *converges only conditionally.*