Name: *Caleb McWhorter — Solutions* **Quiz 13: Power Series**
MAT 296 **Fall 2017**

Problem 1: Determine whether the following series converges or diverges. Be sure to justify your answer.

$$\sum_{n=1}^{\infty} \frac{2ne^n}{n!}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{2(n+1)e^{n+1}}{(n+1)!} \cdot \frac{n!}{2ne^n} \right| = \lim_{n\to\infty} \left| \frac{2n+2}{2n} \cdot \frac{e^{n+1}}{e^n} \cdot \frac{n!}{(n+1)!} \right| = \lim_{n\to\infty} \left| \frac{n+1}{n} \cdot \frac{e}{1} \cdot \frac{\cancel{n!}}{(n+1)\cancel{n!}} \right| = 0 < 1$$

Therefore, the series $\sum_{n=1}^{\infty} \frac{2ne^n}{n!}$ *converges absolutely by the Ratio Test.*

Problem 2: Determine whether the following series converges or diverges. Be sure to justify your answer.

$$\sum_{n=1}^{\infty} \left(\frac{n+5}{5n+1} \right)^n$$

$$\lim_{n\to\infty} \left| \left(\frac{n+5}{5n+1} \right)^n \right|^{1/n} = \lim_{n\to\infty} \frac{n+5}{5n+1} = \frac{1}{5} < 1$$

Therefore, $\sum_{n=1}^{\infty} \left(\frac{n+5}{5n+1} \right)^n$ *converges absolutely by the Root Test.*

Problem 3: Determine the center, interval and radius of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{2^n(x+1)^n}{n}$$

Clearly, the center of this series is $x = -1$.

$$\lim_{n\to\infty} \left| \frac{2^{n+1}(x+1)^{n+1}}{n+1} \cdot \frac{n}{2^n(x+1)^n} \right| = \lim_{n\to\infty} \left| \frac{2^{n+1}}{2^n} \cdot \frac{n}{n+1} \cdot \frac{(x+1)^{n+1}}{(x+1)^n} \right| = |2(x+1)| < 1$$

<u>OR</u>

$$\lim_{n\to\infty} \left| \frac{2^n(x+1)^n}{n} \right|^{1/n} = \lim_{n\to\infty} \frac{|2(x+1)|}{\sqrt[n]{n}} = |2(x+1)| < 1$$

We then have radius of convergence

$$R = \frac{-1/2 - (-3/2)}{2} = \frac{1}{2}$$

Then we must have

$$\begin{aligned} |2(x+1)| &< 1 \\ -1 < 2(x+1) &< 1 \\ -\frac{1}{2} < x+1 &< \frac{1}{2} \\ -\frac{3}{2} < x &< -\frac{1}{2} \end{aligned}$$

If $x = -\frac{1}{2}$*:*

$$\sum_{n=1}^{\infty} \frac{2^n(-1/2+1)^n}{n} = \sum_{n=1}^{\infty} \frac{2^n \cdot 1/2^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

This series clearly diverges by the p-series test.

If $x = -\frac{3}{2}$*:*

$$\sum_{n=1}^{\infty} \frac{2^n(-3/2+1)^n}{n} = \sum_{n=1}^{\infty} \frac{2^n(-1/2)^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Since the series is alternating, $\lim n \to \infty \frac{1}{n} = 0$*, and* $\frac{1}{n}$ *is decreasing in 'n', the Alternating Series Test says that this series converges. Therefore, the interval of convergence is*

$$\left[-\frac{3}{2}, -\frac{1}{2}\right)$$