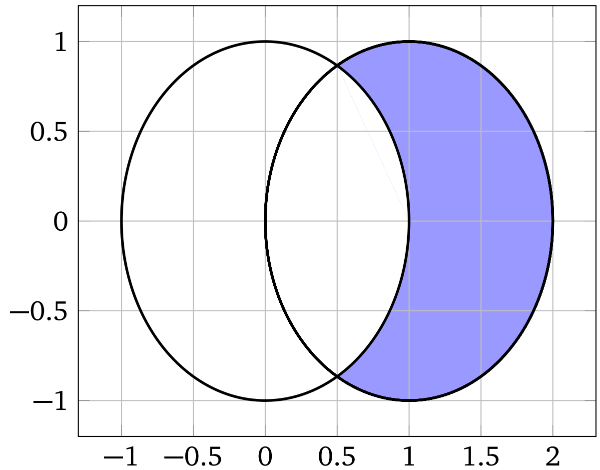
Name: *Caleb McWhorter — Solutions* **Quiz 15: Polar Area**

MAT 296 **Fall 2017**

Problem 1: The curves $r = 1$ and $r = 2\cos\theta$ are plotted below. Find the area of the shaded region.



First, we find the intersection of the two curves:

$$2\cos\theta = 1 \Longrightarrow \cos\theta = \frac{1}{2} \Longrightarrow \theta = \frac{\pi}{3}, -\frac{\pi}{3}$$

Then we have

$$\begin{aligned}
A &= \frac{1}{2}\int_{-\pi/3}^{\pi/3} \left((2\cos\theta)^2 - 1^2\right) d\theta \\
&= \frac{1}{2}\int_{-\pi/3}^{\pi/3} \left(4\cos^2\theta - 1^2\right) d\theta \\
&= \frac{4}{2}\int_{-\pi/3}^{\pi/3} \cos^2\theta \; d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3} 1 \; d\theta \\
&= \frac{2}{2}\int_{-\pi/3}^{\pi/3} 1 + \cos 2\theta \; d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3} 1 \; d\theta \\
&= \int_{-\pi/3}^{\pi/3} 1 \; d\theta + \int_{-\pi/3}^{\pi/3} \cos 2\theta \; d\theta - \frac{1}{2}\int_{-\pi/3}^{\pi/3} 1 \; d\theta \\
&= \int_{-\pi/3}^{\pi/3} \cos 2\theta \; d\theta + \frac{1}{2}\int_{-\pi/3}^{\pi/3} 1 \; d\theta \\
&= \frac{\sin 2\theta}{2}\bigg|_{-\pi/3}^{\pi/3} + \frac{1}{2}\left(2 \cdot \frac{\pi}{3}\right) \\
&= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{-\sqrt{3}}{2}\right) + \frac{\pi}{3} \\
&= \frac{\sqrt{3}}{2} + \frac{\pi}{3} = \frac{2\pi + 3\sqrt{3}}{6}
\end{aligned}$$