Name: *Caleb McWhorter — Solutions* **Quiz 4: Area & Arclength**
MAT 296 **Fall 2017**

Problem 1: Find the arclength of the curve $x(y) = \ln|\sin y|$ for $\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$.

We have $x(y) = \ln|\sin y|$ *so that* $x'(y) = \dfrac{1}{\sin y} \cdot \cos y = \cot y$.

$$L = \int_{\pi/6}^{\pi/3} \sqrt{1+\cot^2 y}\; dy = \int_{\pi/6}^{\pi/3} \sqrt{\csc^2 y}\; dy = \int_{\pi/6}^{\pi/3} \csc y\; dy = \ln|\csc y - \cot y|\Big|_{\pi/6}^{\pi/3}$$

$$= \ln\left|\csc\frac{\pi}{3} - \cot\frac{\pi}{3}\right| - \ln\left|\csc\frac{\pi}{6} - \cot\frac{\pi}{6}\right| = \ln\left|\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right| - \ln\left|2-\sqrt{3}\right| = \ln\left|\frac{1}{\sqrt{3}}\right| - \ln|2-\sqrt{3}|$$

$$= \ln\left|\frac{1/\sqrt{3}}{2-\sqrt{3}}\right| = \ln\left|\frac{1}{2\sqrt{3}-3}\right| = \ln\left|\frac{1}{2\sqrt{3}-3} \cdot \frac{2\sqrt{3}+3}{2\sqrt{3}+3}\right| = \ln\left|\frac{2\sqrt{3}+3}{3}\right| = \ln\left|1+\frac{2}{\sqrt{3}}\right| \approx 0.767652$$

Problem 2: Find the area between the curves $x = y^2 - 1$ and $x = 14 - 2y$.

The curve $x = y^2 - 1$ *is clearly a 'sideways' parabola and the curve* $x = 14 - 2y$ *is a line. It should also be clear, the curve lies 'to the right' of the parabola. We need only find the intersections:*

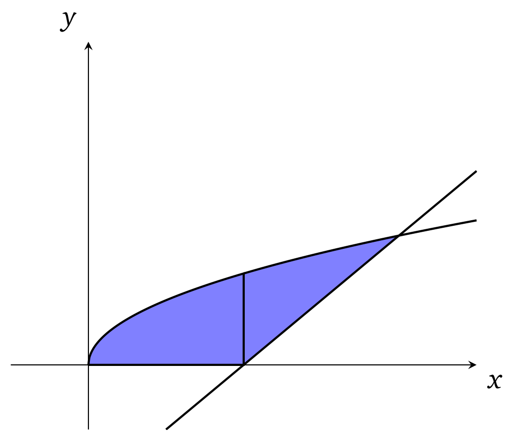
$$\begin{aligned} y^2 - 1 &= 14 - 2y \\ y^2 + 2y - 15 &= 0 \\ (y-3)(y+5) &= 0 \end{aligned}$$

Therefore, we have $y = 3$ *or* $y = -5$. *Then the area between the curves is*

$$\begin{aligned} A = \int_{-5}^{3} (14-2y) - (y^2-1)\; dy &= \int_{-5}^{3} 15 - 2y - y^2\; dy \\ &= \left(15y - y^2 - \frac{1}{3}y^3\right)\Big|_{-5}^{3} \\ &= (15(3) - 3^2 - \frac{3^3}{3}) - (15(-5) - 25 - \frac{(-5)^3}{3}) \\ &= 15(3) - 3^2 - \frac{3^3}{3} + 15(5) + 25 - \frac{5^3}{3} \\ &= 15(3+5) - 9 + 25 - \frac{1}{3}(3^3 + 5^3) \\ &= 120 + 16 - \frac{1}{3}(125 + 27) \\ &= 136 - \frac{152}{3} \\ &= \frac{408}{3} - \frac{152}{3} = \frac{256}{3} \end{aligned}$$

Problem 3: Suppose R is the region in $\mathbb{R}^2$ whose boundary is formed by the curve $y = \sqrt{x}$, the x–axis, and the line $y = x - 2$.

(a) Sketch the region R.



For the following parts, observe $y = \sqrt{x} \Longleftrightarrow x = y^2$ and $y = x - 2 \Longleftrightarrow x = y + 2$.

(b) Set up completely as possible *but do not evaluate* integrals calculating via the Disk/Washer method the volume of the solid formed by rotating the region R about the lines $x = -3$ and $y = 5$.

$$x = -3: \qquad \pi \int_0^2 (3 + (y+2))^2 - (3 + y^2)^2 \, dy$$

$$y = 5: \qquad \pi \int_0^2 5^2 - (5 - \sqrt{x})^2 \, dx + \pi \int_2^4 \big(5 - (x-2)\big)^2 - (5 - \sqrt{x})^2 \, dx$$

(c) Set up completely as possible *but do not evaluate* integrals calculating via the Shell Method the volume of the solid formed by rotating the region R about the lines $x = -3$ and $y = 5$.

$$x = -3: \qquad 2\pi \int_0^2 x\sqrt{x} \, dx + 2\pi \int_2^4 x\big(\sqrt{x} - (x-2)\big) \, dx$$

$$y = 5: \qquad 2\pi \int_0^2 (5 - y)\big((y+2) - y^2\big) \, dy$$