Name: *Caleb McWhorter — Solutions* **Quiz 6: Partial Fractions**
MAT 296 **Fall 2017**

Problem 1: Integrate the following:

$$\int \frac{2x^3 - 13x^2 + 16x - 4}{x - 5}\, dx$$

$$\begin{array}{r@{}l} & \;\; 2x^2 \quad -3x + 1 \\ x - 5 \big) & \;\; 2x^3 - 13x^2 + 16x - 4 \\ & -2x^3 + 10x^2 \\ \hline & \qquad -3x^2 + 16x \\ & \qquad\;\; 3x^2 - 15x \\ \hline & \qquad\qquad\quad x - 4 \\ & \qquad\qquad\; -x + 5 \\ \hline & \qquad\qquad\qquad 1 \end{array}$$

Therefore, we have

$$\int \frac{2x^3 - 13x^2 + 16x - 4}{x - 5}\, dx = \int \left(2x^2 - 3x + 1 + \frac{1}{x-5} \right) dx$$
$$= \frac{2x^3}{3} - \frac{3x^2}{2} + x + \ln|x - 5| + C$$

Problem 2: Integrate the following:

$$\int \frac{5x - 16}{2x^2 + 7x - 4}\, dx$$

$$\frac{5x-16}{2x^2+7x-4} = \frac{5x-16}{(x+4)(2x-1)} = \frac{A}{x+4} + \frac{B}{2x-1} = \frac{A(2x-1) + B(x+4)}{(x+4)(2x-1)} = \frac{(2A+B)x + (-A+4B)}{(x+4)(2x-1)}$$

Therefore by comparison of coefficients of the numerators, we have the equations:

$$\begin{aligned} x: &\quad 5 = 2A + B \\ 1: &\quad -16 = -A + 4B \end{aligned}$$

This system has solution $A = 4$ and $B = -3$. Note that Heaviside's method applies so that one could have found these using

$$A = \frac{5(-4) - 16}{2(-4) - 1} = \frac{-36}{-9} = 4 \qquad\qquad B = \frac{5(1/2) - 16}{1/2 + 4} = \frac{-27/2}{9/2} = -3$$

Then

$$\int \frac{2x - 19}{2x^2 + 7x - 4}\, dx = \int \left(\frac{4}{x+4} - \frac{3}{2x-1} \right) dx = 4 \ln|x+4| - 3 \cdot \frac{1}{2} \ln|2x - 1| + C$$
$$= \ln \left| \frac{(x+4)^4}{\sqrt{(2x-1)^3}} \right| + C$$

Problem 3: Integrate the following:

$$\int \frac{5x^3 - 4x^2 + 3x - 1}{x^2 + x^4}\, dx$$

$$\begin{aligned}
\frac{5x^3 - 4x^2 + 3x - 1}{x^2 + x^4} = \frac{5x^3 - 4x^2 + 3x - 1}{x^2(x^2+1)} &= \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1} \\
&= \frac{Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2}{x^2(x^2+1)} \\
&= \frac{Ax^3 + Ax + Bx^2 + B + Cx^3 + Dx^2}{x^2(x^2+1)} \\
&= \frac{(A+C)x^3 + (B+D)x^2 + Ax + B}{x^2(x^2+1)}
\end{aligned}$$

Comparing coefficients gives the following system of equations:

$$\begin{aligned}
x^3 :& \qquad A + C = 5 \\
x^2 :& \qquad B + D = -4 \\
x :& \qquad A = 3 \\
1 :& \qquad B = -1
\end{aligned}$$

This clearly has solution $A = 3$, $B = -1$, $C = 2$, and $D = -3$. Therefore,

$$\begin{aligned}
\int \frac{5x^3 - 4x^2 + 3x - 1}{x^2 + x^4}\, dx &= \int \left(\frac{3}{x} - \frac{1}{x^2} + \frac{2x - 3}{x^2 + 1} \right) dx \\
&= \int \left(\frac{3}{x} - \frac{1}{x^2} + \frac{2x}{x^2 + 1} - \frac{3}{x^2 + 1} \right) dx \\
&= 3 \ln|x| + \frac{1}{x} + \ln|x^2 + 1| - 3 \arctan x + K
\end{aligned}$$